THE CLOUDY BAG MODEL OF HADRONS

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I. INTRODUCTION

The Cloudy Bag Model[1] is a phenomenological representation of hadronic properties and meson-baryon scattering which includes both quark confinement and pionic cloud effects. The model has a partially conserved axial vector current (obeys PCAC) and in the limit of zero quark mass obeys the current algebra relations of Gell-mann and Levy.[2] It is easy to apply and no large computer calculations are required.

A contrasting approach to hadronic properties is the lattice approximation to quantum chromodynamics (QCD) discussed[3] at this meeting. Impressive results, based on the proposed fundamental theory of the strong interaction, have been obtained. However, the computer time required for these calculations is very great indeed. For example, a recent lattice gauge theory study of the hadronic spectrum carried out at CERN[4] required the equivalent of about 5000 hours of CDC 7600 time. There is nothing wrong with using the computer to learn about hadronic properties. However, we must remember that as nuclear physicists it is our goal to understand the properties of the nucleus. It is very difficult to imagine that studies of such large systems as nuclei could be carried out using present lattice theory techinques. Thus it is necessary that we develop and test simple models of hadrons that can be more easily applied to studies of the nucleus. Even though QCD is THE theory of the strong interaction, phenomenological models are needed. The Cloudy Bag Model is one such approach.

The outline of this talk is as follows:

1. A more detailed explanation of "What is a cloudy bag?".
2. Summary and update of early results.
3. Current algebra properties of the Cloudy Bag Model[5,6] with emphasis on recent studies of mesonic decays.[7]
4. Connection[8] between the axial vector form factor, pion-nucleon vertex function and the Goldberger-Treiman relation.

Let the story begin.

II. WHAT IS A CLOUDY BAG?

Our starting point is the much-discussed MIT bag model[9] which in its simplest version treats the baryon as three non-interacting, relativistic quarks confined in a static, spherical cavity. Although the so-called static cavity approximation violates the Lorentz invariance of the theory, its relativistic nature does provide some advantages. As a theory of massless quarks it might be a step closer to QCD than the constituent quark model. The relativistic Lagrangian formulation allows one to construct conserved currents, so that ultimately (with the inclusion of the pionic cloud) the constraints of current algebra can be satisfied. In addition, the asymptotic freedom and confinement aspects of QCD are implemented in an extremely simple fashion. The MIT bag model leads to generally good (but far from perfect) results for the energies and other properties of the hadronic ground states.

However there is a very significant (to a nuclear physicist, anyway) problem with the MIT bag model. It does not allow for the emission and absorption of a pion by a nucleon. This means that there is no one pion exchange potential (OPEP) between nucleons. The existence of OPEP is perhaps the most solid fact[10] that can be gleaned from all of the extensive analyses of nucleon-nucleon scattering, so that the MIT bag is in direct conflict with experiment. Furthermore, this problem is not restricted to the two nucleon system. All nucleons are identical so that if a pion can be emitted by one nucleon and absorbed by another, it can be emitted and absorbed on the SAME nucleon. This means that nucleons have a pion cloud. The above argument is older than I am, but it is just as correct as ever. Thus the MIT bag lacks a pion cloud that it should have.

There is a more fancy way to say the same thing. Even if the pion were massless in the MIT bag model, the Lagrangian fails to obey the very important property of chiral symmetry. An even more high-brow way to put it is that the model has no SU(2) x SU(2) (or any other) current algebra.

The breaking of chiral symmetry and its repair is intimately connected with the inclusion of pionic effects,[11] so I will discuss this subject in a bit more detail.

II.1 CHIRAL SYMMETRY BREAKING OF THE MIT BAG

The basic point is that quarks, q, carry an axial current $\underset{\sim}{A}^{MIT}$, such that

$$\underset{\sim}{A}_{\mu}^{MIT} = \frac{1}{2}\bar{q}\,\gamma_{\mu}\gamma_{5}\underset{\sim}{\tau}\,q\,\Theta(R-r) \tag{1}$$

where R is the bag radius. With the use of standard MIT bag model solutions one can easily show that

$$\hat{r}\cdot\underset{\sim}{\vec{A}}\,\Big|_{r=R} \neq 0 \tag{2}$$

that $\hat{r}\cdot\underset{\sim}{\vec{A}}$ does not vanish at the surface of the bag even though the surface value of the normal component of the standard current, $\bar{q}\gamma_{\mu}q$, is zero is possible only through the relativistic nature of the quark wave functions. The upper and lower components allow for two different values of the currents at the bag surface. The consequence of Eq. (2) is that axial flux leaves the bag. When this is combined with the fact that in a static bag the axial charge is constant one is immediately led to the result:

$$\partial^{\mu}\underset{\sim}{A}_{\mu}^{MIT} \neq 0 \;. \tag{3}$$

The axial current is not conserved. Eq. (3) has nothing to do with the pion mass; even when the pion mass is set to zero Eq. (3) holds.

II.2 CHIRAL SYMMETRY AND THE BAG

The solution of this problem was obtained by Brown & Rho and others.[12] One simply includes the effects of a pion field, $\underset{\sim}{\phi}$. Since the pion carries an axial current (that is responsible for the process $\pi \rightarrow \mu\nu$, one can obtain a conserved current by allowing a quark which hits the surface to transfer its axial current to a pion that can exist outside, Fig. 1. Thus if one regards the baryon as a system of three quarks in a bag that is surrounded by an extended pion cloud, no axial flux is lost. This is because the expectation value of $\underset{\sim}{\phi}$ vanishes for distances far from the bag center. Thus one can indeed construct

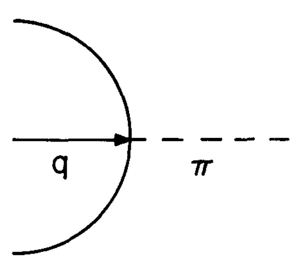

Figure 1. Pionic mechanism for enforcing conservation of axial vector current.

bag models for which

$$\partial_\mu A^\mu = 0 \tag{4}$$

with $m_\pi = 0$.

II.3 DO PIONS MATTER?

Pionic effects must be included to allow for the existence of OPEP and the resulting baryonic pion cloud, as well as to obtain a conserved axial vector current (and PCAC for non-zero values of the pion mass). Furthermore with pions one can show that the necessary current commutation relations are satisfied.[13] Thus pions must be included. The only operative question is: How much do pions change the properties of baryons?

In order to answer the above question consider a lowest-order perturbation theory calculation of the nucleonic expectation value of the pion number operator, N_π. As shown in Fig. 2, the process is of second order in the pion-nucleon coupling constant, which via the Goldberger-Treiman relation is proportional to 1/f, with f the pion decay constant $\cong$ 93 MeV. Since the expectation value of the pion number operator must be dimensionless, and the only other significant parameter is the bag radius, R, we must have

$$\langle N_\pi \rangle \propto \frac{1}{f_\pi^2 R^2} \quad . \tag{5}$$

The coefficient of proportionality is about 1/14. The point of Eq. (5) is that for a small bag there are lots of pions, but for a large bag, there are not so many.

This brings us to the fundamental philosophical point behind the

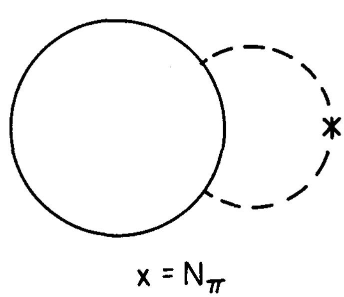

Figure 2. Expectation value of the pion number operator. The pion is represented as a dashed line. The pion number operator N_π is represented by the cross.

Cloudy Bag Model. It is our view that QCD forces are responsible for confinement phenomena. Although pions supply needed corrections, they do not cause confinement. This leads us to a picture of the nucleon in which the bag has a fairly large radius (close to that of the MIT bag, but perhaps a bit smaller), and the pionic effects are amenable to a perturbation-theoretic treatment. The implementation of this view is the subject of the next sub-section.

II.4 TECHNIQUE

The Lagrangian density of the Cloudy Bag Model is given by the expression

$$\mathcal{L}_{CBM}(x) = [i\bar{q}\partial q - B]\,\Theta_V - \frac{\Delta_s}{2}\,\bar{q}\,\exp(i\underset{\sim}{\tau}\cdot\underset{\sim}{\phi}\gamma_5/f)\,q$$
$$-\frac{1}{2}\,D_\mu\underset{\sim}{\phi}\cdot D^\mu\underset{\sim}{\phi} + \frac{m_\pi^2}{2}\,\phi^2 \qquad (6)$$

The quark field operator is q, B is the positive phenomenological energy density of the bag, Θ_V is a function that is unity for positions inside the bag and vanishes outside, and Δ_s is a surface delta function. The pion field operator is $\underset{\sim}{\phi}$ and, the original MIT bag model is regained if $\underset{\sim}{\phi}$ is set to zero. The pion-quark interaction occurs at the bag surface and the pion kinetic energy and mass are included in the last two terms of the right-hand-side of Eq. (6).

To treat the pion as an elementary field, as in Eq. (6) is to make a long wavelength approximation since effects of the pion's finite extent are neglected. One justification for this is that the pionic size (which is less than its measured r.m.s. charge radius[14]) is much smaller than its Compton wavelength. We do not deny that the pion has a finite size, but wish to use the above model only in low momentum transfer situations for which the size should be irrelevant.

In writing the pion kinetic energy as in Eq. (6) we allow the pion to exist inside as well as outside the bag. Pions inside the bag can be generated, for example, by residual non-confining forces between a quark and an anti-quark. Since these forces are particularly strong and attractive in the $q\bar{q}$ channel with the quantum numbers of the pion,[14,15] a correlated $q\bar{q}$ pair inside the bag that looks pretty much like a pion could exist. In any case, there is no fundamental or phenomenological reason to exclude pions from the interior of the bag as is done in Ref. 11. If m_π is set to zero $\mathcal{L}_{CBM}$ has a conserved axial current. Furthermore by allowing the pions to penetrate the bag, one avoids[16] a well-known problem[17] of obtaining too large a value of g.

As stated above we regard the bag radius, R, to be fairly large and the pion field to be fairly small. Thus we use a perturbative quantum field theoretic treatment of pionic effects. The zero'th order result is just the usual MIT bag model solution, so that the nice features of that model are incorporated from the beginning. Furthermore the terms of Eq. (6) non-linear in $\underset{\sim}{\phi}$ contain effects of pion loops. The finite pion size is expected to be significant in reducing the values of such diagrams, so that we believe that it is reasonable and sensible to neglect non-linear pionic effects. Since $\underset{\sim}{\phi}$ is small and non-linearities are ignored we obtain the approximations:

$$\exp\,(i\tau\cdot\phi\gamma_5/f) \Rightarrow 1 + i\underset{\sim}{\tau}\cdot\phi\gamma_5/f_\pi$$

$$D_\mu\phi \Rightarrow \partial_\mu\underset{\sim}{\phi} \qquad (7)$$

The resulting Lagrangian is simple and still obeys PCAC.

Another approximation is to allow quarks to exist only in the $s_{1/2}$ single particle MIT bag state of lowest energy. If one includes effects of the pion's finite extent in both the relative space and time variables,[18] one can show that effects of quarks in excited states are very small indeed.

Another procedural matter to be discussed is the treatment of the bag radius, R. It is determined from calculations of masses in the MIT bag model. However, computed values of masses are insensitive to the value of R. This is especially true if one considers uncertainties in some unmentioned parameters that go into the mass calculations. In our studies of pionic effects in calculations of various observables we regard R as a basically free parameter.

Values in the range between 0.8 fm and 1.1 fm result in good agreement with a variety of observables.

It is to be noted that pionic field effects do not dynamically influence the bag radius in our approach. This is undoubtedly an approximation. However, the inclusion of the finite size of the pion as well as the procedure of obtaining the self-energy as the solution of the appropriate transcendental equation (Eq. (3.19) of Ref. 19) leads to the result[13] that effects of the pionic pressure terms on the bag radius are fairly small.

There is one very serious problem with the MIT bag model that cannot be handled by including pionic effects. By requiring that the bag be centered at a point fixed in space one introduces a variety of spurious center-of-mass effects. This results in a theoretical uncertainty of at least ten percent in the calculated value of any observable.

III. SUMMARY OF EARLY CLOUDY BAG MODEL RESULTS

In this section some of the first calculations are reviewed. Subjects covered include pion-nucleon scattering in the (3,3) resonance region, static properties of nucleons, and the magnetic moment of the sigma hyperon, Σ^- .

III.1 PION-NUCLEON SCATTERING IN THE (3,3) RESONANCE REGION

It is well-known that there is a very large peak in pion-nucleon cross-section that occurs when the pion kinetic energy is about 200 MeV. This peak is referred to as the delta. One long standing question concerned the exact nature of the delta. In the Chew-Low-Wick[20] theory the delta is a composite system of a pion and a nucleon. The attraction is provided by the crossed Born term, which is iterated to produce a transition matrix in accord with the experimental data. (The iteration of the crossed Born term is the solution of the Low equation if one neglects the left-hand pion-nucleon cut.[21]) A more modern view[22] is that the delta is a three-quark state that is weakly coupled to the pion-nucleon continuum. The Cloudy Bag Model has both sets of terms. The procedure is to vary R, the only free parameter, to obtain the best fit to pion-nucleon scattering data. This gives a first handle on the value of R. Furthermore one also can examine the solution to see which of the Chew-Low series or the three-quark delta is the more important. This enables us to answer the question: What is a delta?

The answer is shown in Fig. 3. (More details are available in Ref. 19.) We see that the best fit is obtained with R = 0.82 fm. This was one of the earliest Cloudy Bag Model determinations of the bag radius. Later work by Théberge[23,24] showed that for $0.8\text{ fm} < R < 1.1\text{ fm}$ one could obtain an essentially equivalent fit. The next step is to study the relative importance of the crossed Born series and the elementary delta terms by setting either the $\pi NN(f_{\pi NN})$ or the $\pi N\Delta$, vertex function ($f_{\pi N\Delta}$) equal to zero. If the $f_{\pi N\Delta}$ is set to zero, the resonance disappears as shown by the dotted curve. On the other hand, if $f_{\pi NN} = 0$ the resonance peak still exists, it moves up by a relatively small amount of about 50 MeV, dash-dotted line. Thus the three-quark delta terms are dominant.

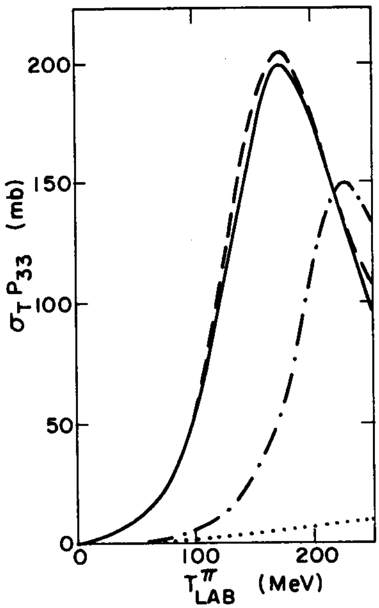

Figure 3. Best fit in the cloudy bag model (dashed curve) to the experimental P33 total cross section (solid). The dash-dotted (dotted) line shows the effect of arbitrarily setting the πNN ($\pi N\Delta$) coupling constant to zero.

III.2 STATIC PROPERTIES OF THE NUCLEON

Values of the bag radius such that are consistent with pion-nucleon scattering in the (3,3) resonance region. The spectrum of hadronic ground states (except the pion) can also be reproduced with such values.[25,26] Thus having constrained the bag radius, we obtain a parameter-free model which can be tested by computing various nucleonic properties.

Let's discuss two of these in a qualitative fashion. The first is the neutron's charge distribution, which is known to be non-zero. For example, see Galster et al.[27] In the Cloudy Bag Model the neutron sometimes consists of a proton and a π^- cloud. Since the positive charge is confined by the bag radius, while the π^- can extend outside by a distance of about its Compton wavelength, one expects the mean-squared-charge radius to be negative. This is indeed the experimental result. Within the framework of the MIT bag model the non-zero charge distribution of the neutron can be caused by gluonic exchange effects. However, a variety of calculations show that this effect is far too small.[28]

Another quantity of interest is the nucleon magnetic moment. The current of the moving charged pions gives rise to a contribution to the nucleonic magnetic moment. The electromagnetic properties of the nucleon are obtained from the Feynman diagrams of Fig. 4.

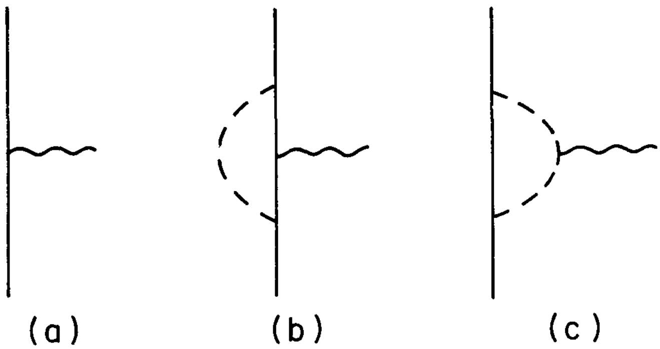

Figure 4. Electromagnetic properties of the nucleon. The baryon is a solid line, the pion a dashed line and the photon a wiggly line. The intermediate baryonic states are either the nucleon or the delta.

We note that the perturbation theory approach to calculating these quantities is valid[24,25] for bag radii such that $R \gtrsim 0.7$ fm.

A typical set of results is shown below in the table. The Cloudy Bag Model, the MIT Bag Model and the experimental values of various observables are compared. The bag radius is taken to be 1.0 fm to facilitate the comparison between our work and that of the MIT group.

OBSERVABLE	CLOUDY BAG	EXPERIMENT	MIT BAG
CHARGE RADII			
$\langle r^2_{ch}\rangle_p^{1/2}$	0.81 fm	0.83 fm	0.73 fm
$\langle r^2_{ch}\rangle_n$	-0.13 fm^2	-0.12 fm^2	0.00 fm
MAGNETIC MOMENTS			
μ_p	2.65 (2.4)	2.79	2.3 (1.9)
μ_n	-2.05 (-1.9)	-1.91	-1.6 (-1.4)

Table 1. NUCLEONIC PROPERTIES. The bag radius is 1 fm. The magnetic moments are given in units of nucleon magnetons. The terms in parentheses are computed without the Donoghue-Johnson effect.

In every case inclusion of pionic effects leads to improvement in the comparison with experiment. The result for the charge radius of the neutron is especially nice. The calculated magnetic moments include the Donoghue-Johnson[29] correction for the center-of-mass motion. Values shown in parentheses omit this questionable effect.

Using again R = 1.0 fm we can determine the composition of the physical nucleonic state. It is a nucleon bag 71% of the time, nucleon bag plus a pion 17% of the time, and a delta bag plus a pion 12% of the time. Thus in our picture the average number of pions in a nucleon is 0.29. There are not a great number of pions.

III.3 MAGNETIC MOMENTS OF THE BARYON OCTET-Σ^- MOMENT

Théberge and Thomas[30] studied the ground state baryon octet which consists of P,n Λ, $\Sigma^{\pm,0}$, $\Xi^{0,-}$ and examined the pion cloud effects. Pions do not interact with the strange quarks so that, generally speaking, pionic effects are small for strange hadrons.

However the Σ^- is an interesting exception. As noted by Pilkuhn and Eeg[31] there are two possible terms involving the emission of a π^-, as shown in Fig. 5. Thus pionic effects are about twice as large as usual for the Σ^-, so that it provides an interesting testing ground for various bag models.

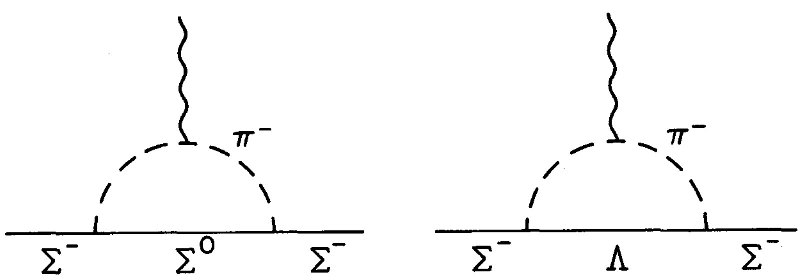

Figure 5. Magnetic moment contributions to the Σ^-.

The theoretical values of the Σ^- magnetic moment in nuclear magnetons for the MIT bag, cloudy bag and little bag are listed below:

THEORY	μ_{Σ^-}
Cloudy Bag	-1.08
MIT Bag	-0.81
Little Bag[32]	-0.58

For a long time the experimental situation[33] was cloudy, with an old atom experiment giving the result -1.41 ± 0.27 and a Σ^- beam experiment finding -0.89 ± 0.14. A new generation of Σ^- atom experiments performed by the William and Mary group at Brookhaven give -1.09 ± 0.03.

The agreement with the Cloudy Bag Model calculation is rather good. The early Little Bag prediction seems to be completely off. (Both of these calculations were made before the William and Mary experiment.)

Having briefly reviewed the earlier work, we turn to newer results.

IV. CURRENT ALGEBRA AND THE CLOUDY BAG MODEL

In this section some of the consequences that current algebra presents are discussed. It has long been known that the use of current algebra enables one to predict many relationships between observables (see e.g. Ref. 35) but in the Cloudy Bag Model one has a new situation. The current algebra predictions are maintained, but are embedded in a Lagrangian which enables one to predict absolute values of observables.

The beginning of the section deals with a version of the Cloudy Bag Lagrangian in which its current algebra aspects are more explicit than in Eq. (6). As a result there are some new applications to mesonic (as opposed to baryonic) systems obtained and discussed in section III.2.

IV.1 THOMAS TRANSFORMATION OF THE CLOUDY BAG MODEL

Although the Cloudy Bag Lagrangian of Eq. (6) has conserved axial and vector currents and obeys[13] the SU(2) x SU(2) current algebra it was difficult to identify the explicit terms and dynamics which led to the well-known current algebra results. For example, it was not clear how Eq. (6) could be used to obtain s-wave pion-nucleon scattering.

This problem was solved by Thomas[5] (see also Ref. 6) who obtained a transformation on the quark field operators which not only made the current algebra results more explicit, but also simplified the form of the Lagrangian and improved its convergence properties.

The basic idea can be understood by considering the surface delta term of Eq. (6)

$$\bar{q} \exp (i\underset{\sim}{\tau}\cdot\underset{\sim}{\phi}\gamma_5/f)\; q\; \Delta_s \qquad (8)$$

Thomas (using a technique similar to that of Weinberg[35]) redefined the quark fields according to

$$q_W = \exp\,(i\underset{\sim}{\tau}\cdot\underset{\sim}{\phi}\gamma_5/2f)\;q \tag{9}$$

so that the surface term of Eq. (8) becomes

$$\bar{q}_W\; q_W\; \Delta_s \tag{10}$$

From Eq. (10) it is clear that the surface boundary conditions of the Cloudy Bag Model are much simpler when expressed in terms q_W. Indeed, the linear boundary condition of the MIT bag model holds for q_W, which is a nice simplification.

Of course, the transformation of Eq. (9) must be applied to the entire Cloudy Bag Model Lagrangian. For the quark kinetic energy term we have that

$$i\bar{q}\partial\!\!\!/ q\Theta_V \Rightarrow i\bar{q}_W\; \exp(i\underset{\sim}{\tau}\cdot\underset{\sim}{\phi}\gamma_5/2f)\; \partial\!\!\!/\; \exp(i\underset{\sim}{\tau}\cdot\underset{\sim}{\phi}\gamma_5/2f)q_W\Theta_V \tag{11}$$

An exact expression for the right-hand-side of Eq. (11) has been given by Thomas. It is my intention here to exhibit the essential features of Eq. (11) and, for this it is sufficient to make an expansion in powers of $1/f$, keeping terms to order $1/f^2$. The result is

$$i\bar{q}\partial\!\!\!/ q\Theta_V \Rightarrow i\bar{q}_W\partial\!\!\!/ q_W\Theta_V + \frac{1}{2f}\,\bar{q}_W\gamma_\mu\gamma_5\underset{\sim}{\tau} q_W\partial^\mu\underset{\sim}{\phi}\Theta_V$$

$$- \frac{1}{2f_\pi^2}\,\bar{q}_W\,\frac{\gamma_\mu\underset{\sim}{\tau}}{2}\,q_W\cdot(\underset{\sim}{\phi}\times\partial^\mu\underset{\sim}{\phi})\;\;\Theta_V \tag{12}$$

In the term that is independent of $\underset{\sim}{\phi}$, one simply recovers the quark kinetic energy. The next term is linear in $\underset{\sim}{\phi}$ and is responsible for the familiar terms that cause emission and absorption of a single pion. It is of a volume pseudovector form, so that at first glance one might believe that the pion-nucleon and pion-nucleon-delta vertex functions are different than the ones obtained from Eq. (6). However, this is not the case. For the systems of interest here, we have that

$$\int \frac{d^3x}{2f}\,\bar{q}_W\gamma_\mu\gamma_5\underset{\sim}{\tau}\;q_W\;\;\partial^\mu\;\;\phi\;\;\Theta_V = \frac{i}{2f}\int d^3x\;\bar{q}_W\gamma_5\underset{\sim}{\tau} q_W\cdot\underset{\sim}{\phi}\;\;\Delta_s \tag{13}$$

the relation (13) holds for systems in which all quarks are in the same orbital.

The proof is very simple so that I reproduce it. Observe first that the term with $\mu = 0$, vanishes by virtue of parity conservation. The remaining term involving the gradient of the pion wave function is simplified by an integration by parts. The use of field equation satisfied by the quarks shows that terms involving gradients of the quark (or adjoint) wave functions cancels as long as the q_W and the $\bar{q}_W$ refer to quarks of the same energy. The remaining term is of a surface delta form. The last step is to use the linear boundary condition to obtain the form (13).

The term involving $\underset{\sim}{\phi}\times\partial^{\mu}\underset{\sim}{\phi}$ is the one of particular interest to Thomas.[6] It is responsible for s-wave pion baryon scattering. In particular, it yields the well-known result of Tomazowa[36] and Weinberg. As shown in the next section it also has some nice consequences regarding the various decay channels of the ρ meson.

IV.2 CLOUDY BAG MODEL AND MESONIC PROPERTIES

I'd like to discuss some recent work done by Paul Singer and myself[7] in which the Cloudy Bag Model is applied in the understanding of various decay widths of the vector mesons.

Start by considering the rho meson which decays via the strong interaction to two pions and via the electromagnetic into a e^+e^- pair. The very same term that is responsible for the s-wave pion nucleon scattering also explains both of these rho meson decay widths and the relationship between them. The relevant matrix element is repeated below (we leave out the subscript W in this subsection.)

$$\frac{1}{2f^2} \langle\rho|\bar{q}\ \frac{\gamma_\mu}{2}\ \underset{\sim}{\tau}\ q\ \Theta_V|0\rangle(\underset{\sim}{\phi}\times\partial_\mu\underset{\sim}{\phi}) \tag{14}$$

The quark isovector current connects the rho meson with the vacuum and the $\underset{\sim}{\phi}\times\partial_\mu\underset{\sim}{\phi}$ term creates two pions. The result is the transition matrix element for the decay $\rho\rightarrow\pi\pi$ which is defined as $f_{\rho\pi\pi}$. The matrix element connecting the rho to the vacuum also appears in the decay $\rho\rightarrow e^+e^-$. It is defined as m_ρ^2/f_ρ. By taking the matrix element of the term of (14) between a physical rho state and a two-pion state one obtains the result

$$\frac{1}{2f^2}\ \frac{m_\rho^{\ 2}}{f_\rho} = f_{\rho\pi\pi} \tag{15}$$

This is the well-known KFSR relation which is well-satisfied experimentally.

This analysis of the Cloudy Bag Model Lagrangian shows that it does satisfy the KFSR relation. But one can go even further, since the bag model allows one to compute the constant, f_ρ. It is a straightforward procedure to evaluate the matrix element $\langle\rho|\bar{q}\gamma_{\mu}\underset{\sim}{\tau}q|0\rangle_{\text{Bag}}$. However f_ρ is defined in terms of the physical rho meson which is an eigenstate of momentum. To bridge the gap it is necessary to relate the rho bag state to momentum eigenstates via a wave packet expansion. We use Wong's[38] version of the Peierls-Yoccoz[39] projection procedure. The calculation is described in Ref. 7, so we only present the results here. It is true that the procedure used is very crude (as is any of the current attempts to fix the static cavity approximation's lack of Lorentz invariance). However the severity of the problem is mitigated somewhat by the relatively large mass of the rho meson. Indeed were the pion mass equal to the rho mass the discrepancy between the Donoghue-Johnson[29] and Wong computations of the pion decay constant, f, would vanish.

The calculation yields a value $f_\rho = 5.4$ which is in good agreement with the value 5.09 ± 0.31 extracted from the partial decay width for the rho to decay into an electron-positron pair.[40] The next step is to compute $f_{\rho\pi\pi}$ and the resulting width for $\rho \to \pi\pi$. We use the KFSR relation Eq. (15) along with the calculated value of f_ρ to get $f_{\rho\pi\pi} = 6.3$. By squaring this number and multiplying it by the appropriate phase space factors one obtains $\Gamma(\rho \to \pi\pi) = 164$ MeV which is in excellent agreement with the experimental value of 154 ± 5 MeV.

Thus the decay widths for the processes $\rho \to \pi\pi$ and $\rho \to e^+e^-$ as well as the algebraic relationship between their amplitudes are well-understood in the Cloudy Bag Model.

Another application to mesonic systems involves calculating the vertex functions for absorption or emission of a single pion. In particular we may compute the coupling constants for the processes $\omega \to \rho\pi$, $K^* \to K\pi$ and $K^* \to K^*\pi$ using the term linear in $\underset{\sim}{\phi}$ of the Lagrangian of Eq. (6) or the one of Ref. (5). One uses standard bag wave functions, with bag radii of Ref. 26, to compute the necessary matrix elements. For example we compute

$$\frac{1}{2f}\langle K\pi|\int d^3x\,\bar{q}\,\gamma_5\underset{\sim}{\tau}\cdot\underset{\sim}{\phi}q\Delta_s|K^*\rangle \tag{16}$$

The resulting coupling constants when expressed in terms of dimensionless units[7] are:

$$f^{\omega\rho\pi} = 10.3,\; f^{K^*K\pi} = 5.8,\; f^{K^*K^*\pi} = 10.2 \tag{17}$$

Experimental information is available from the decays $\omega \to \pi\pi\pi$ and $K^* \to K\pi$ so that one can extract the values

$$f^{\omega\rho\pi}_{EXPT} = 12.0 \pm 0.4 \qquad f^{K^*K\pi}_{EXPT} + 6.5 \pm 0.2 \tag{18}$$

A comparison of the result for $f^{K^*K^*\pi}$ with experiment is difficult. There is only an experimental upper limit available,[40] $\Gamma_{K^*}(K\pi\pi)<36$ keV. Our value is consistent with the SU(3) prediction $f^{K^*K^*\pi^0} = \frac{\sqrt{3}}{2} f^{\omega\rho\pi}$ (corrected in our model by an SU(3) breaking factor M_{K^*}/M_{ω}), which gives[41] for this decay a rate approximately equal to the present upper limit.

A comparison between Eqs. (17) and (18) shows that the Cloudy Bag Model provides an excellent description of the single pion emission vertices. When combined with the results for the decays of the rho meson, we see that both one and two pion emission vertices are well explained.

Another application, still in progress, involves computing the electromagnetic decays of a vector meson into a pseudoscalar meson and a single photon. These transitions occur via M1 transitions. For example, consider the processes $K^{*+} \to K^+\gamma$ and $K^* \to K\gamma$. SU(3) symmetry predicts that the width for the former process is only one-fourth of the latter, whereas the experimental values for the two widths are almost equal.[42] The effects of the strange quark mass are not sufficient to account for the rates.[42] However, including the pion cloud contribution shown in Fig. 6 does lead to an explanation of the data.[43] The present case is an exception to the general rule that pionic effects are small for strange systems because of the relatively unimportant contribution of the ordinary term (Fig. 6a) in the process $K^{*+} \to K^+\gamma$. Thus it appears that the Cloudy Bag Model will provide a reasonable explanation of these M1 transitions.

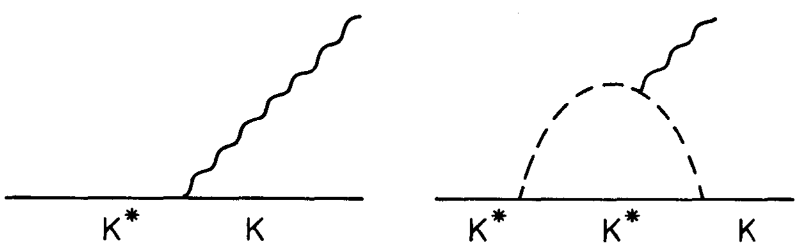

Figure 6. M1 Decays of the K Mesons. a) photon-quark interaction term. b) pion cloud contribution. The wavy line represents the photon, and the dashed line the pion.

V. THE AXIAL FORM FACTOR OF THE NUCLEON, PION-NUCLEON VERTEX FUNCTION AND THE GOLDBERGER-TREIMAN (GT) RELATION

In this section the axial form factors $g_A(q^2)$, vertex function $g_{\pi NN}(q^2)$ and the relation between them are discussed. An application is made to the understanding of the error in the GT relation.

As mentioned above there has been considerable discussion of the calculation of the axial charge of the nucleon in various chiral bag models. However little attention has been given to the axial-vector form-factor, $g_A(q^2)$, which is known from measurements of muon production cross-sections ($\nu_\mu + n \rightarrow \mu^- + p$) and pion electroproduction. Phenomenologically the best fit is a dipole with mass $m_A = 0.95 \pm 0.14$ GeV. In fact Amaldi et all[44] quote an error of ± 0.07 GeV from the neutrino reaction, and ± 0.14 GeV from electroproduction. We take the more conservative error in order to improve the overlap with the pion production data from the ZGS, which yielded values of m_A between 1.0 and 1.4 GeV.[45] Of course these data are most sensitive to the low $-q^2$ behavior, which can be parametrized as

$$g_A(q^2)/g_A(0) = 1 - q^2 r_A^2/6 \ , \tag{19}$$

with $r_A = 0.72 \pm 0.12$ fm.

In the Cloudy Bag Model the pion field is allowed to penetrate the interior of the MIT bag. The appropriate expression for the axial-vector current density is

$$\vec{A}_i(\underset{\sim}{x}) = 1/2\ \bar{q}(\underset{\sim}{x})\ \vec{\gamma}\gamma_5\tau_i\ q(\underset{\sim}{x})\ \theta(R-r) - f_\pi\ \vec{\nabla}\ \phi_i(\underset{\sim}{x}) \tag{20}$$

For this model $g_A(q^2)$ is entirely determined as the (coefficient of the term proportional to $\sigma\tau_i$ of the) expectation value of the Fourier transform of the first term in Eq. (20) in the physical nucleon.

There is also a term proportional to $\vec{\sigma}\cdot\hat{q}\hat{q}$ in the above Fourier transform, but it is <u>not</u> part of $g_A(q^2)$. One interesting thing about the Cloudy Bag Model is that the $\vec{\nabla}\underset{\sim}{\phi}$ term of Eq. (20) does not contribute to $g_A(q^2)$. This is because the pion is allowed to penetrate the bag so that a plane wave expansion of $\underset{\sim}{\phi}$ is justified. Upon taking the Fourier transform of (20) the $\vec{\nabla}\underset{\sim}{\phi}$ term becomes one proportional to $\hat{q}$ which does not contribute to g_A. Thus in the Cloudy Bag as in the MIT Bag the value of $g_A(q^2=0) \equiv g_A$ is 1.09. This is in contrast with other bag models[11] which by excluding pions from the bag interior obtain an enhanced value of g_A of $\frac{3}{2}$ (1.09) which is much too big. (The difference between 1.09 and the experimental value of 1.26 can be obtained in a variety of ways. See, for example, Ref. 46.

Taking the expectation value of the Fourier transform of Eq. (20) leads to the result

$$g_A^{CBM}(q^2)/g_A^{CBM}(0) = 1.5\ j_0^{-2}(\omega) \int_0^R dr\ r^2 \Big\{[j_0^2\ (\tfrac{\omega r}{R}) - j_1^2\ (\tfrac{\omega r}{R})]\ j_0(qr) + 2\ j_1^2\ (\tfrac{\omega r}{R})\ j_1(qR)/(qR)\Big\}. \quad (21)$$

As usual, $\omega = 2.04$ is the eigenfrequency for massless quarks.

Expanding Eq. (21) to order q^2 for comparison with Eq. (19) we find $r_A^{CBM} = 0.62$ R, and hence $R = 1.16 \pm 0.20$ fm. (Of course we must add a caution that we have also omitted corrections arising from possible spurious centre of mass motion and recoil.) The result of these considerations is that the axial vector form factor is very soft. It is clear that better data would be much appreciated.

Another property of the nucleon which has been intensely discussed in the context of chiral bag models is the form-factor for coupling to a pion, $g_{\pi NN}(q^2)$. As shown in Eq. (21), in such models $g_A(q^2)$ arises from an integral over the bag volume. On the other hand, $g_{\pi NN}(q^2)$ is associated with the surface of the bag, and one might intuitively expect it to fall more rapidly with q^2. In the CBM this is certainly the case, because the form-factor there is

$$\begin{aligned} g_{\pi NN}(q^2) &= 3j_1(qR)/(qR) \\ &= 1 - q^2(0.6R^2)/6\ . \end{aligned} \quad (22)$$

In analogy with the treatment of r_A we can define r_π through the relation

$$g_{\pi NN}(q^2) \equiv 1 - q^2 r_\pi^2/6 + 0(q^4). \quad (23)$$

This gives $r_\pi = 0.77$ R. This should be compared with the value of r_A(=0.62R) calculated from Eq. (21). That is, in the CBM (r_A/r_π) equals 0.80.

Thus the πNN form factor is even softer than the axial form factor. This is because the π-quark interaction is at the surface with $r = R$, whereas the axial vector interaction occurs throughout the bag volume where $r \leq R$.

It is useful to observe that the form factor $3j_1(qR)/(qR)$ has a very simple interpretation in the Cloudy Bag Model. The pion is in a plane wave with angular momentum one. Hence one has a j_1 function. The pion interacts with quarks only at the bag surface so that the argument of the spherical Bessel function must be qR. The factor 3/qR is included to obtain the normalization $g_{\pi NN}(q^2 = 0) = 1$.

At this stage we recall the infamous discrepancy in the Goldberger Treiman relation

$$5.97 \pm 0.03 = g_A(0)\ m_N \neq g_{\pi NN}\ f_\pi = 6.36 \pm 0.10\ , \tag{24}$$

where we have used the measured value of the NNπ coupling constant[47] ($g^2/4\pi = 14.3 \pm 0.4$) at the nucleon pole ($q^2 = -m_\pi^2$). On the other hand the derivation of Eq. (24) requires that one should use $g_{\pi NN}$ at $q^2 = 0$. It is amusing to ask what radius (r_π^{GT}) would give enough variation between $q^2 = -m_\pi^2$ and $q^2 = 0$ to resolve all of this discrepancy. Clearly that would be $(r_\pi^{G-T})^2\ m_\pi^2/6$ equal to $6.1 \pm 1.7\%$, or

$$r_\pi^{G-T} = 0.86 \pm 0.12 \text{ fm.} \tag{25}$$

Using the experimental value for r_A given in Eq. (1), and the CBM ratio $r_A/r_\pi = 0.80$, we find $r_\pi = 0.90 \pm 0.15$ fm. (We hope that the use of the ratio of these quantities will reduce the unknown error associated with centre of mass motion.)

Thus, within the rather large experimental errors, the variation of the NN_π form-factor with q^2 in the Cloudy Bag Model is enough to remove all of the Goldberger-Treiman discrepancy.

Let us summarize briefly. We have calculated the q^2-dependence of the axial-vector form-factor of the nucleon, and the NNπ vertex function, using the Cloudy Bag Model. These calculations use a static bag, without c.m. or recoil corrections. All of these approximations should be improved in future work. Nevertheless our central result is very simple, and probably correct independent of these approximations. That is, the NNπ form-factor is considerably softer than the axial form-factor. (The corresponding r.m.s. radius is at least 25% larger.) If one then uses the experimental axial form-factor, the q^2 dependence of the resulting NNπ form-factor is enough to entirely remove the Goldberger-Treiman discrepancy -- within the rather large experimental uncertainty. At this stage we might remark that the NNπ form-factor obtained in this fashion is rather soft. Clearly it would be very valuable to have more precise data on $g_A(q^2)$, which is of fundamental importance in chiral bag models. Finally we must point out that there may well be additional contributions to the Goldberger-Treiman discrepancy -- see, for example, Refs. 48 - 50. Our point is merely that given the rather large experimental errors at present, these additional corrections are not essential.

VI. SUMMARY

The Cloudy Bag Model is a phenomenological treatment of hadronic structure. Quarks confined in an MIT bag are surrounded by a cloud of pions. The pionic effects are incorporated in such a way that the properties of chiral symmetry and the corresponding consistency with current algebra relations are maintained. As a result one is able to describe a great many phenomena. The root mean square charge radii of nucleons; magnetic moments of baryons; pion-nucleon scattering; strong and electromagnetic decays of vector mesons; and g_A are all computed in agreement with experiment. The discrepancy in the Goldberger-Treiman relation is also explained. To achieve these results a bag radius in the range 0.8 fm to 1.1 fm is required. Thus fairly large bags are needed. This is in agreement with the considerations of the MIT and many other bag models. The interesting implication, in my opinion, is that with such bag sizes nucleons overlap in nuclei, and that one should expect to encounter nuclear phenomena that depends on quark degrees of freedom.

I would like to thank my colleagues, A. W. Thomas and S. Théberge. Without them there would be no Cloudy Bag Model. I'd also like to thank my collaborators on the more recent projects discussed in this talk: G. A. Crawford, P. Guichon, M. Morgan and P. Singer. This work was partially supported by the U. S. Department of Energy.

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