CENTRE OF MASS CORRECTION TO BAG MODEL

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Introduction

In the MIT bag model [1], the calculation of any property of hadron is bound to the static sferical cavity approximation in which bag is well localised in space. By the uncertainity principle, it follows that such a state can not be an eigenstate of the total linear momentum. The expectation value of the total momentum squared in this state is different from zero, in other words, the centre of mass is in motion. Nevertheless in early applications of the bag model, the ground state of quarks in the cavity was treated as representing hadron at rest. The results for magnetic moments,axial coupling constant, etc, calculated in such approximation contained contributions from the centre of mass motion. There have been many attempts to improve the original model and to calculate appropriate corrections [2,3,4,5,6,7]. The results obtained by various authors are very different; not only magnitude, but also the signs of some corrections differ from paper to paper. This concerns particularly the corrections to magnetic moments, but other corrections are not free from ambiguity either.

The various approaches leading to contradictory results differ by their specific assumptions. In order to clarify the situation, and to understand the source of discrepancy I shall present an approach to the centre of mass (CM) corrections based essentially on only one approximate assumption, which is however common to all calculations we are talking about. I will restrain from making additional, more specific assumptions until I get formulae showing unambiguously sign and order of magnitude of the appropriate correction. Only then I shall be more specific and apply these results to making detailed fits to the data.

The plan of my talk is following: Chapter I will be devoted to the discussion of basic assumption that the bag state at rest represents, at a given time, the superposition of free physical particle states with different linear momenta. In Chapter II I shall present formulae for corrected static parameters of baryons. Chapter III contains comparison of these results with particle data.

The three chapters correspond to three papers [8,9,10] written in co-laboration with J. Bartelski, Z. Ryzak from Warsaw University, and L. Mankiewicz, S. Tatur from N. Copernicus Astronomical Centre of Polish Academy of Sciences.

I. The wave packet approximation

The MIT bag model wave function has two characteristic features: 1) It is localised in space, and 2) It is a product of independent quark wave functions. The second property limits accuracy with which localised state may be represented as a wave packet of states of *the same internal motion*. It is well known that in nonrelativistic case, only for harmonic oscilator the product wave function $\Pi u_i(r_i)$ of independent particles may be cast in the "wave packet" form:

$$\prod_i u_i(r_i) = \int \Phi(P)\{e^{iPR_{CM}}\cdot\Psi(\text{int. coord.})\}d^3P = F(R_{CM})\cdot\Psi(\text{int.coord.}) \quad (1)$$

For two particles, for example, the following identity holds:

$$e^{-m_1 r_1^2}\cdot e^{-m_2 r_2^2} = e^{-MR^2}\cdot e^{-\ r^2} = N\int e^{-P^2/4M}\{e^{iPR}\cdot e^{-\mu r^2}\}d^3P \quad (2)$$

where μ is the rtduced mass, M is the total mass, r and R are the relative and CM positions respectively.

For any other functions, in particular for "bag-like" functions u_i localized within sharply defined boundary, representation (1) is not valid. However one may still wonder how good the representation (1) could be as an approximation. It is very difficult to answer this question for relativistic case, but we may get some feeling what the real situation is by investigating the simpler nonrelativistic case.

We choose an example of two nonrelativistic particles of equal masses in a one dimensional square well potential. It has the merit of being the simplest possible case, and at the same time the most vulnerable for discrepancy (the more degrees of freedom, the less severe the problem of CM motion). Sharp boundaries seem also to be as much contrasting with smoothness of the oscillator wave function as possible. Mathematically, we are looking for two functions F(R) and $\Psi(r)$ which minimize the deviation:

$$\delta\int |u(R+r/2)u(R-r/2) - F(R)\Psi(r)|^2 dR\, dr = 0 \quad (3)$$

for given $u(x)=N\sin kx$.

Solution of the above problem leads in a natural way to the expansion:

$$u(r_1)\,u(r_2) = \sum\sqrt{\lambda}_i F_i \Psi_i\,, \quad \sum \lambda_i = 1 \tag{4}$$

where $\{F_i\}$ and $\{\Psi_i\}$ are certain orthonormal sets defined by Eq(3). The left hand side of Eq(4) is well approximated by only one term if the largest eigenvalue λ_{max} is close to 1. In the opposite case, e.g. for $\lambda_{max} \cong 0.5$, we would say that the approximate representation of independent particles wave function $u_i(r_i)$ by the wave packet of fixed internal motion is completely unjustified.

In practice it turns out, that for the case under consideration

$$\lambda_{max} \cong 0.98$$

what is very close to 1. The obtained result is obviously not a proof of applicability of the formula analogous to Eq(1) for the relativistic bag model, but it certainly provides some credibility to such an approximate assumption.

In what follows we shall therefore assume:

$$|B,j_z\rangle = \int d^3p\Phi(|\vec{p}|)\,|\vec{p},s_z\rangle \;; \qquad j_z = s_z \tag{5}$$

Letter B refers to bag, $|\vec{p},s_z\rangle$ is the state of physical particle with quantum numbers of the corresponding bag state. The wave packet on the right hand side of Eq(4) is assumed to have the same parity and angular momentum as the bag state on the left. This makes $\Phi(|\vec{p}|)$ a function of $|\vec{p}|$ only, what simplify calculations significantly. The Eq(5) is in fact *the most general one consistent with the neglect of particles mixing.*

II. Corrections for CM motion

The typical developpment from now, which seems to be one of the sources of discrepancies among published results, consists of the following: Translating Eq(5) by $\vec{a}$ one obtains

$$e^{i\hat{\vec{P}}\vec{a}}|B\rangle_{\vec{0}} = |B\rangle_{\vec{a}} = \int d^3p\Phi(p)e^{i\vec{p}\vec{a}}|\vec{p}\rangle \tag{6}$$

which after integration over $\vec{a}$ gives:

$$|\vec{p}\rangle = \frac{1}{(2\pi)^3\Phi(p)}\int d^3a e^{-i\vec{p}\vec{a}}|B\rangle_{\vec{a}} \tag{7}$$

Taking appropriate scalar product one also obtains:

$$|\Phi(p)|^2 = \frac{1}{(2\pi)^3}\int d^3a \; {}_0\langle B|B\rangle_a \, e^{-ipa} \tag{8}$$

With help of Eq(7) we may now express any matrix element between physical particles through the matrix elements between bag states $|B\rangle$. In this way all expressions for physical matrix elements will contain in the integrand a quantity:

$${}_0\langle B|J|B\rangle_a \tag{9}$$

The matrix element (9) is then calculated by integrating quark wave functions over the overlaping part of two bags. Such procedure implicitely assume, that scalar product of two,partly overlaping, empty bags is 1:

$${}_0\langle EB|EB\rangle_a = 1$$

The above assumption introduces uncontrolable errors. It is impossible to calculate the empty bag matrix element because nonquark degrees of freedom (responsible for confinement) are not actually quantized in the bag model.

In these circumstances, for not to introduce uncontrolable assumptions I shall consider only equations containing matrix elements of currents (known to be built out of quark only) between states with *identical empty bags*. Contrary to a prejudice one may meet frequently, I claim that such a matrix element (without one of the bags being boosted or shifted with respect to the other) gives useful informations about physical matrix element, not only for momentum transfer equal zero, but also in some vicinity of this point, allowing calculations of magnetic moments, g_A, and electromagnetic (as well as weak) squared radii.

Making use of Eq(5), we may immediately write down:

$$\langle B,s_z'|J(\vec{x},0)|B,s_z\rangle = \int d^3p d^3p' \Phi^*(p')\Phi(p) e^{i(\vec{p}-\vec{p}')\vec{x}} \langle s_z',\vec{p}'|J(0,0)|\vec{p},s_z\rangle \tag{10}$$

Without boosting bags, Eq(10) contains already on the right hand side various fourmomentum transfers $(E(\vec{p}')-E(\vec{p}),\vec{p}'-\vec{p})$ between physical hadrons. This is enough for the appearence of physical formfactors which we shall try to extract from the above relation. In this sense Eq(10) takes into account both "CM correction" and "recoil correction" which need not be separated in any way. As long as one does not propose better connection between bag states and particle states than that expressed by Eq(5), one can not avoid Eq(10).

Multiplying Eq(10) by exp(ikx) and integrating over x we get

$$\int d^3x e^{ikx} \langle B|J(x)|B\rangle = \int d^3p \Phi^*(p+k)\Phi(p)\langle p+k|J(0)|p\rangle \tag{11}$$

(the spin indeces have been supressed). Formally the Eq(11) is an identity with respect to $\vec{k}$, but in practice it is as approximate as the basic assumption about wave packet representation given by Eq(5).

Differentiating Eq(11) with respect to k (once or twice) and putting k=0, we may generate relations involving measurable quantities (on the right hand side) and bag model quantities (on the other side).

For illustration let us consider this equation for $J = J^o_{em}$ taken between nucleon states with $s_z = s'_z$ and differentiated twice with respect to k. For k=0 on the left hand side we obtain:

$$-\int x^2 \langle B\ J^o_{em}(\vec{x},0)\ B\rangle d^3x \tag{12}$$

which may immediately be calculated ($=-0.53R_B^2$) and is called static charge squared radius. On the right hand side there appear several terms. One of them is connected with the argument of formfactors:

$$F_i(k_\mu k^\mu) = F_i\{(\sqrt{m^2+(\vec{p}+\vec{k})^2} - \sqrt{m^2+\vec{p}^2})^2 - \vec{k}^2\} \cong F_i\{-\vec{k}^2(1-\frac{1}{3}\frac{p^2}{E^2})\} \tag{13}$$

leading to:

$$\left.\frac{dF_i}{d\vec{k}^2}\right|_{\vec{k}^2=0} = -(1-\frac{1}{3}\frac{p^2}{E^2})F'_i(0) \tag{14}$$

$F'(0)$ is directly related to physical electromagnetic (electric and magnetic, which are slightly different) squared radii, while factor $(1 - p^2/3E^2)$ averaged with $|\Phi(p)|^2$ is one of the typical corrections. It is of course, just the Lorentz contraction.

Another important term comes from differentiating $\Phi^*(p+k)$ in Eq(11) It is:

$$\langle R^2\rangle = -\int \Phi^*(p)\nabla^2\Phi(p)d^3p \tag{15}$$

With nonrelativistic profile, this would be just the average square of CM position; in more general case the physical interpretation of this quantity may be obscure (recall difficulties with the definition of position operator for relativistic particle), but this need not concerne us, because Eq(15) defines uniquely this quantity.

In this way we have obtained the following formulae:

$$g_A = g_A^{st\ bag} \frac{3}{1+2\langle\frac{M}{E}\rangle} \tag{16}$$

$$\mu = \mu^{st\ bag} \frac{3}{1+\langle\frac{M}{E}\rangle+\langle\frac{M^2}{E^2}\rangle} + \frac{M_p}{M} Q \frac{1-\langle\frac{M}{E}\rangle}{1+\langle\frac{M}{E}\rangle+\langle\frac{M^2}{E^2}\rangle} \tag{17}$$

$$\langle r^2\rangle = \frac{\langle r^2\rangle^{st\ bag} - Q\langle R^2\rangle}{\frac{2}{3}+\frac{1}{3}\langle\frac{M^2}{E^2}\rangle} + \text{calculable but numerically negligible terms} \tag{18}$$

All averages have to be calculated with profile $\Phi(p)$, e.g.

$$\langle\frac{M}{E}\rangle = \int |\Phi(|p|)|^2 \frac{M}{\sqrt{M^2+p^2}} d^3p \tag{19}$$

Till now we have made no assumption in addition to the basic one (Eq(5)), which can not be avoided however, if a given bag state is connected with one sort of physical particles. Having assumed so little, we are not able to calculate electroweak properties to the end; the profile (p) is not yet specified. Nevertheless, even without this specification Eqs (16), (17), (18) allow already to draw very definite conclusions. For example, whatever the profile $\Phi(|p|)$ is, we have always

$$\langle\frac{M}{E}\rangle < 1 , \qquad \langle\frac{M^2}{E^2}\rangle < 1$$

and consequently

$$\mu > \mu^{static\ bag} \tag{20}$$

what is not a trivial result in view of disagreement in the literature concerning the sign of $\mu-\mu^{st\ bag}$.

For numerical quantitative applications, Eqs (16), (17) and (18) are also very convenient, because they are in fact very unsensitive to the details of the distribution of the momentum defined by the profile $\Phi(\vec{p})$. this allows us to calculate reliably all corrections with few additional but very plausible assumptions.

First of all we shall assume, that averages like $<\frac{M}{E}>$ and $<M^2/E^2>$ are not independent. Formally, if $\Phi(\vec{p})$ is not specified, average $<M/E>$ and $<M^2/E^2>$ can take arbitrary, unrelated values. However for smooth distributions, containing no extreme velocities, functions like M/E or M^2/E^2 fulfill, with great accuracy, the naive relation:

$$f(<v>) = <f(v)> \tag{21}$$

The above observation reduces the information contained in $\Phi(\vec{p})$ to practically one parameter. More attention is needed in estimation of $<R^2>$. Because of the form of Eq(15), we have immediately the "uncertainity relation"

$$<R^2><p^2> \geqq \frac{9}{4} \tag{22}$$

In order to estimate $<R^2>$ we take the equality sign

$$<R^2> \cong \frac{9}{4<p^2>} \tag{23}$$

This result wold be exact for Gaussian profile. Introducing the average boost factor γ,

$$\frac{1}{\gamma} = <\frac{M}{E}> \tag{24}$$

we have

$$<\frac{M}{E}> = \frac{1}{\gamma}$$

$$\begin{aligned} <\frac{M^2}{E^2}> &= \frac{1}{\gamma^2} \\ <R^2> &= \frac{9}{4M^2(\gamma^2-1)} \end{aligned} \tag{25}$$

$$<p^2> = M^2(\gamma^2-1)$$

Finally one has to specify γ. The simplest choice is to take only contribution from quarks and calculate $<p^2>$.

$$\langle p^2\rangle = M^2(\gamma^2-1) = \frac{3x_0^2}{R^2} \qquad (26)$$

for nonstrange baryons, and similarly for other hadrons. This is the most arbitrry assumption in our approach. However, whatever the further developpment of the bag theory is, the order of magnitude of $\langle p^2\rangle$ will rather not change. And because our formulae (16), (17), (18) do not contain cancellation, and depend smoothly on $\langle p^2\rangle$, the arbitrariness of the assumption (26) is not very dangerous to the reliability of obtained results.

Before going to applications, let me quote one particular result, namely that for $\langle R^2\rangle$, in the case of massless quarks in the nucleon:

$$\langle R^2\rangle = \frac{9(x_0-1)}{2(2x_0^3-2x_0^2+4x_0-3)}\langle r^2\rangle^{st\ bag} = 0.34\langle r^2\rangle^{st\ bag} \qquad (27)$$

which is surprisingly close to the nonrelativistic, classical value $\frac{1}{3}$.

Applications

The MIT bag model contains several number of adjustable parameters. Inclusion of any correction,like CM correction, needs always refitting all of the parameters. The usual strategy is to choose certain particles and calculate parameters from their masses, and then to predict remaining masses and electroweak properties. The choice is of course arbitrary and the results somehow accidental. The most objective choice of parameters would consist in taking all calculable properties and fitting parameters by some kind of least square method. Because,however there is no obvious prescription how to weight various contributions of different physical dimensions,this will not be a very good procedure either.

We have decided to take all masses of the ground state multiplets and to look for model parameters which minimize the function

$$\chi^2(m_u,m_s,Z,B,\alpha_s) = \sum(M_{exp}-M_{bag})^2 \qquad (28)$$

The pion is not included in the above sum, because no stable solution could be found in this case(before CM corrections had been included). The minimalization procedure applied to the original, uncorrected version of the model, improves the overall agreement of masses which is already surprisingly good in the fit of de Grande et al.[1].The new parameters do not differ to much from the old one. They are compared in Table 1.

Table 1

orig. MIT	$m_u=0$	$m_s=279$ MeV	$\alpha_s=2.2$	Z=1.84	$B^{\frac{1}{4}}=146$ MeV	$\sqrt{\chi^2}=23$ MeV
χ^2 method CM uncorr.	12MeV	283 MeV	2.04	2.01	149 MeV	16 MeV
$M=\sqrt{E_B^2-\langle p^2\rangle}$	10MeV	286 MeV	1.68	0.86	144 MeV	16 MeV

The next step was to fit masses corrected for CM motion according to the formula

$$M = \sqrt{E_B^2 - \langle p^2\rangle} \tag{29}$$

The stability equation was taken as the minimum of E_B . Resulting parameters are shown in the third row of Table 1. The changes of Z and α_s go in theoretically desirable direction, but the accuracy stays at the same level ($\langle\delta M\rangle=16$ MeV). For electroweak properties however, the improvement is enormous! Results are shown in Table2

Table 2

		μ_{exp}	μ_B	μ_B (CM corrected)
magnetic moments	P	2.79	1.82	2.53
	N	-1.91	-1.21	-1.64
	Λ	-0.61	-0.50	-0.61
	Σ^+	2.37	1.88	2.39
	Σ^-	-1.18	-0.64	-0.93
	Ξ^0	-1.25	-1.08	-1.33
	Ξ^-	-0.69	-0.45	-0.58
	g_A	1.25	1.10	1.25
	$\langle r^2\rangle_P$	$(0.84\text{fm})^2$	$(0.69)^2$	$(0.67)^2$
	$\langle r^2\rangle_N$	$-(0.34\text{fm})^2$	0	0

The additional merit of taking into account the CM motion is,that the pion may now be treated at the same footing as other hadrons.

Repeating fit with pion contribution included into χ^2 we obtain the following results:

m_u=3MeV, m_s=283MeV, α_s=1.6, Z=0.75, $B^{\frac{1}{4}}$=142MeV, $<\delta M>$=19MeV. With the last value for average discrepancy, the mass of the pion comes out equal 125 MeV to be compared with experimental 138 MeV.

Summarizing the above results we may say, that simple MIT model supplemented by CM corrections in the way explained in Chapter II describes masses of hadrons (including pion) and their electroweak properties very well. One may even say, that it reproduces these properties (particularly masses) too well. The agreement is so good that it seems not to leave any room for corrections from the pion cloud, which must not be omitted if chiral symmetry is to be included into the bag model [11, 12, 13, 14, 15, 16, 17].

In view of the necessity of refitting everything each time something new is added to the model, this suspition has to be checked by explicit calculation. If the pion cloud contribution to the energy is taken in its simplest form

$$E_\pi = -\frac{1}{f_\pi^2 R^3} f(m_\pi R, m_u R) \sum_{\substack{nonstr\\quarks}} (\vec{\sigma}\tilde{\tau})^i (\vec{\sigma}\tilde{\tau})^j \qquad (29)$$

no reasonable, stable fit to masses is possible. The main reason is the large pion cloud contribution to the pressure balance equation which typically exceeds the gluon one by a factor of three. One possible way out is to exclude pion cloud contribution from pressure equilibrium equation and retain it only in the expression for energy, as some authors do [18]. There is no convincing argument in favor of this procedure; we have tried another, purely phenomenological approach. It consists in replacing the bare value of the quark-pion coupling constant $g^o=1/2f_\pi$ in the expression (29) by an effective value g^{eff} which is then fitted together with B, α_s, etc. Such a replacement is motivated by a recent indication [19] that finite size of the pion may lower effectively the overall strength of the pion field in the bag model. The fitted value which emerges is $g^{eff}=0.57g^o$.

Conclusions

I want to conclude with three simple statements:

i) CM corrections are calculable and by themselves bring the MIT model to the very good agreement with measured static properties of hadrons

ii) $\bar{q}\,q$ pion state is at the same footing as other hadrons

iii) There is no room for full chiral pion cloud contribution to the energy. Effective coupling constant giving good fit is $0.57g^{O}$.

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