DYNAMICS OF THE SOLITON BAG

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The MIT bag[1] was one of the earliest and most successful models of QCD, imposing confinement and including perturbative gluon interactions. An evolution of the MIT bag came with tne introduction of the chiral[2] and cloudy[3] bags, which treat pions as elementary particles. (The pion is an "anomalously" light hadron, and in a chirally invariant QCD should emerge as the massless Goldstone boson.) Non-relativistic potential models also have had remarkable success, and have the advantage of being amenable to dynamic calculations.

As a model of QCD, the soliton model proposed by Friedberg and Lee[4,5] is particularly attractive. It is based on a covariant field theory and is sufficiently general so that, for certain limiting cases of the adjustable parameters, it can describe either the MIT or SLAC (string) bags. The confinement mechanism appears as a dynamic field. This allows non-static processes, such as bag oscillations and bag collisions, to be calculated utilizing tne well-developed techniques of nuclear many-body theory.

In the soliton model, the (effective) Hamiltonian density is

$$\mathcal{H} = \mathcal{H}_q + \mathcal{H}_\sigma + \mathcal{H}_{q\sigma} + \mathcal{H}_G \quad +\text{(counter terms, Higgs fields,etc)}, \qquad (1)$$

where the individual terms have the followıng interpretation:

$\mathcal{H}_q = \Sigma_f\, \psi_f^+\, (\vec{\alpha}\cdot\vec{p} + \beta m_f)\, \psi_f$ describes the quarks as Dirac particles of mass m_f, where f is the flavor. We take $m_u = m_d = 0$.

$\mathcal{H}_\sigma = \frac{1}{2}\,(\pi_\sigma^2 + |\vec{\nabla}\sigma|^2) + U(\sigma)$ describes the scalar soliton field σ, which represents the complex structure of the vacuum, arising from virtual gluons and quark-pairs interacting among themselves. The non-linearity of the soliton field enters through the self-interaction function (see Fig. 1)

$$U(\sigma) = \frac{1}{2} a \sigma^2 + \frac{1}{3!} b \sigma^3 + \frac{1}{4!} c \sigma^4 + B. \tag{2}$$

The polynomial terminates in fourth order to ensure renormalizability. $U(0) = B$ is to be identified with the "bag constant" or volume energy density of a cavity. With a suitable adjustment of the constants, the function has two minima, one at $\sigma = 0$, and another, lower minimum at $\sigma = \sigma_v$. The physical vacuum corresponds to the second minimum, and the constant B is chosen so that $U(\sigma_v) = 0$.

The quarks interact with the soliton field through the term $\mathcal{H}_{q\sigma} = g\bar{\psi}\sigma\psi$. In the presence of (real) quarks, the sum $U(\sigma) + g\bar{\psi}\sigma\psi$ may have a minimum (depending on the parameters) near $\sigma = 0$ (the perturbative vacuum). This leads to a cavity in the σ-field, which is called the "bag."

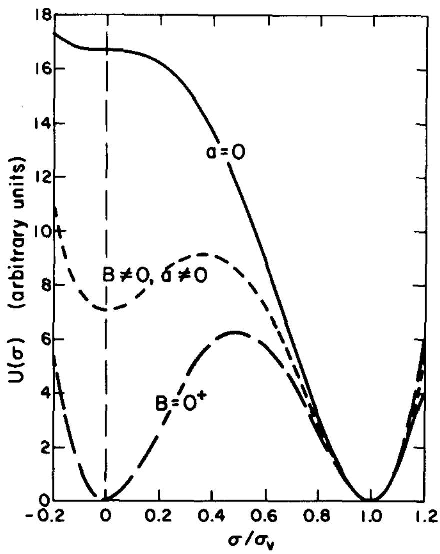

Fig. 1. Three forms for $U(\sigma)$.

Color gluon fields are introduced as in QCD, except that they interact with the soliton field through a dielectric function $\kappa(\sigma)$, chosen such that $\kappa(0) = 1$ and $\kappa(\sigma_v) = 0$. A convenient (but not unique) form is

$$\kappa(\sigma) = (\sigma/\sigma_v - 1)^2 . \tag{3}$$

The magnetic susceptibility is $\mu = \kappa^{-1}$. The requirements on κ yield color confinement. This can be seen easily if one keeps only terms linear in the gluon field. Then Gauss's law gives

$$\vec{\nabla}\cdot\vec{D}^c + \rho^c, \tag{4}$$

where c is the color index. If the total color charge does not vanish within some finite cavity, the D-field will fall off as r^{-2} as $r \rightarrow \infty$, and the color electric energy in medium

$$\frac{1}{2}\int d^3r\ D^2/\kappa(r) \tag{5}$$

will be infinite because $\kappa\rightarrow 0$ (exponentially in the model) as $r\rightarrow\infty$.

As long as one calculates diagrams only through order of one gluon exchange, there is no problem of double counting: the soliton field represents at least two-gluon structures. If higher order diagrams are calculated, the coefficients in the effective Lagrangian must be readjusted at each stage to compensate.

There are five parameters in the model: a, b, c, g and α_s. The first four involve only the soliton field and quarks; α_s is the quark-gluon coupling constant. The following key data may be used to help fix these parameters:

Soliton-quark parameters.

From the presentations of Drs. Engels and Schierholz earlier (see also refs. 6 and 7), it is encouraging to note that we now have available from lattice gauge theory (LGT) calculations the following pieces of data.

(1) The bag constant B = (220+20) MeV/fm^3 . This is considerably larger than the MIT value[8] of 57 MeV/fm^3 .

(2) m_{GB} = (720+40) MeV, the mass of the glueball state (0^{++}). We identify this state with an excitation of the pure soliton field, such that $U''(\sigma_V) = m^2_{GB}$.

The errors quoted in (1) and (2) above are presumably statistical only, and do not account for systematic effects of the lattice size or the omission of dynamical interactions with quarks.

From experiment we select the subset

(3) The mean of the nucleon and delta masses, $\bar{m} = (m_N + m_\Delta)/2$ = 1.087 GeV.

(4) The proton size $\langle r_p^2 \rangle^{1/2}$ = 0.83 fm.

(5) The proton magnetic moment, μ_p = 2.7925.

(6) The ratio of the axial to vector coupling constants, g_A/g_V = 1.25.

The strong coupling constant α_s. This is a "running" constant, and is evaluated for the regime of hadronic sizes. To fix it, we may utilize

(7) The delta-nucleon mass difference, $m_\Delta - m_N = 297$ MeV.

(8) The "string constant, which is the coefficient of the linear term in the potential between massive quarks, such as charmed quarks. This has been fit phenomenologically to the charmonium spectrum using a non-relativistic potential model[9] and has also been obtained in LGT calculations[7] ; the value is $t \cong 1$ GeV/fm. In the MIT model, $t = (32 \pi\alpha_s B/3)^{1/2}$. It is also calculable in the soliton model.

Before this workshop, we did not have data (1) and (2) available, so the results presented here are for fits of the parameters to the remaining data.

There is much more data to fit once the parameters are determined. These include hadronic spectra and resonances, decay widths, various reactions (especially nucleon-nucleon scattering), etc. There are many more experimental data of high and low quality, but the data from LGT calculations are still few and of uncertain accuracy.

In adjusting the model parameters, the fitting is dependent on the level of sophistication of the model calculations. Whenever more diagrams are included, one must readjust the model parameters. For example, if we were able to calculate all gluon interactions exactly, then presumably the effects of the soliton field would disappear completely.

We quote results here for the soliton model calculated through the mean field approximation plus realistic recoil corrections . The latter are calculable in the soliton model, since the "cavity" carries both energy and momentum. By measuring the proton form factor relative to its center-of-energy, the size is seen to be smaller than that measured with respect to the bag center; hence a larger bag is needed to fit experiment than would be the case for a static bag. In calculating hadronic masses, the total momentum term is subtracted according to the relativistic relation

$$m^2 = E^2 - P^2 . \qquad (6)$$

The strong coupling constant has been fixed using one gluon exchange calculations for the $m_\Delta - m_N$ mass difference based on the MIT bag model (scaled for size).

Pion effects have not yet been included explicitly. Although these could be calculated in the manner of the chiral,[2] cloudy,[3] or chiral-soliton[12] bag models, our philosophy is that they should emerge in the soliton model as $q\bar{q}$ pairs interacting with the σ and gluon fields. The pion bag mass has been calculated in the MIT model[8] to be small. Recoil effects reduce it further. The pion cloud surrounding a nucleon should be describable as a surface oscillation of the nucleon σ-cavity accompanied by virtual $q\bar{q}$ pairs. Many reactions involving pions are certainly easier to handle mathematically in the chiral and cloudy bag models, however.

Through the order of calculations stated above, we find a "reasonable" parameter set which gives a moderately good fit to experimental data:

$$a = 43.84,\ b = -7\,184,\ c= 180\,000,\ g = 25,\ \alpha_s = 2.49. \qquad (7)$$

The structure of the soltion field and the quark wave function for the nucleon are shown in Fig. 2. Other hadronic properties are summarized in Table 1.

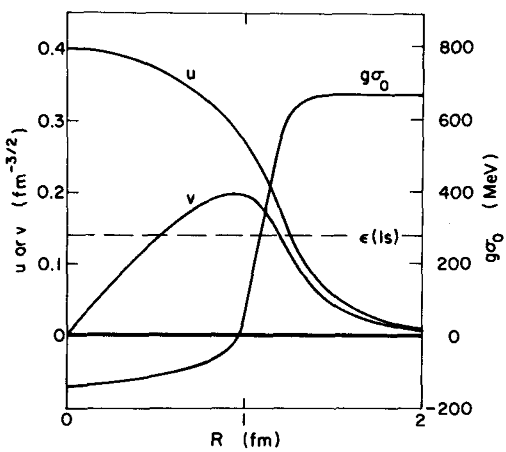

Fig. 2. Numerical results for the parameter set given in text above.

The method of generator coordinates (GCM) is being applied to large amplitude bag dynamics. Consider a parameter or set of parameters α which describe the static configuration of a system of quarks and the soliton field and let $|\alpha,n\rangle$ denote a set of basis states which is complete for any α. A method of obtaining these basis states is described below. The GCM state vector is written

$$|\Psi\rangle = \sum_n \int \phi_n(\alpha)\,|\,\alpha, n\rangle\, d\alpha \tag{8}$$

Since the set is complete for each α, the expansion is overcomplete. In practice, this causes no problem since the sum is truncated to a small number of terms. In what follows, we consider only a single term and suppress n; actually, several configurations may be required. The generalization is straightforward.

The weight function $\phi(\alpha)$ is obtained by minimizing the expectation value of the Hamiltonian $\langle\Psi|H|\Psi\rangle$ subject to the normalization constraint $\langle\Psi|\Psi\rangle = 1$:

$$\delta\langle\Psi|H-E|\psi\rangle/\delta\phi^*(\alpha) = 0 = \int d\alpha'\langle\alpha|H-E|\alpha'\rangle\phi(\alpha') \ . \tag{9}$$

This is the basic GCM integral equation for $\phi(\alpha)$. Depending upon whether the spectrum is discrete or continuous, it is either an eigenvalue or a scattering equation. Although we can work with the integral equation, it is instructive to consider the approximate differential equation which it satisfies. For a system which has well developed collective-motion, we expect $\langle\alpha|H-E|\alpha'\rangle$ to fall off rapidly as a function of $\alpha'-\alpha$. (This is a criterion for α to be a good collective coordinate.) Then we can expand $\phi(\alpha')$ in a Taylor series about $\alpha'=\alpha$, to obtain

$$\begin{aligned} &\left[\int d\alpha'\langle\alpha|H-E|\alpha'\rangle\right]\phi(\alpha) \\ &+\left[\int d\alpha'\langle\alpha|H-E|\alpha'\rangle(\alpha'-\alpha)\right] d\phi/d\alpha \\ &+\left[\int d\alpha'\langle\alpha|H-E|\alpha'\rangle(\alpha'-\alpha)^2\right]\tfrac{1}{2}\, d^2\phi/d\alpha^2 \\ &+ \ldots\ldots = 0. \end{aligned} \tag{10}$$

With the definition $H_n(\alpha) = \int d\alpha'\langle\alpha|H|\alpha'\rangle(\alpha'-\alpha)^n$

and $N_n(\alpha) = \int d\alpha'\langle\alpha|\alpha'\rangle(\alpha'-\alpha)^n$, this assumes the simpler form

$$(N_0^{-1}H_0-E)\phi + N_0^{-1}(H_1-EN_1)\phi' + N_0^{-1}(H_2-EN_2)\tfrac{1}{2}\,\phi'' + \ldots\ldots = 0. \tag{11}$$

If we terminate the Taylor series with ϕ'', we have the beginning of a Schroedinger-type equation. Note, however, that H_0/N_0 is <u>not</u> the conventional mean field term for the potential energy, which is usually taken to be $\langle\alpha|\, H\,|\alpha\rangle$. There is a term in ϕ', and the coefficient of ϕ'' is α-dependent. By well-known techniques we can eliminate the ϕ' term or, better, recast the equation into the Hermitean form

$$[-\frac{d}{d\alpha}\ \frac{1}{2B(\alpha)}\ \frac{d}{d\alpha} + V(\alpha) - E]\tilde{\phi}(\alpha) = 0\ . \tag{12}$$

It is an "elementary" exercise to relate $(\tilde{\phi},V,B)$ to (ϕ,H_n,N_n). Later we define α such that $\alpha \rightarrow r$ as $r \rightarrow \infty$, where r is (say) the separation of two bags. However, $V(\alpha)$ has no simple interpretation as a potential until $B(\alpha)$ is determined!

Following Mosel,[13] we now introduce a change of variable from α to x such that

$$x = X(\alpha) = \int^{\alpha}[B(\alpha')/\mu]^{\frac{1}{2}}d\alpha'\ , \tag{13a}$$

$$\tilde{\phi}(\alpha) = f(\alpha)\ \psi(x)\ , \tag{13b}$$

$$f(x) - \text{const.}\quad [\mu B(\alpha)]^{\frac{1}{4}}\ , \tag{13c}$$

where $\mu = M/2$ is the reduced nucleon mass. Then Eq. (12) becomes

$$(-\frac{1}{2\mu}\frac{d^2}{dx^2} + V + V_1 - E)\ \psi(x) = 0\ , \tag{14}$$

with

$$V_1 = \frac{1}{8}\frac{B''}{B^2} - \frac{1}{3}\frac{(B')^2}{B^3} \tag{15}$$

Since we choose α so that for separated bags $\alpha \rightarrow r$ (the bag separation) this is consistent with $B \rightarrow \mu$, $x \rightarrow \alpha$.

We determine $|\alpha\rangle$ by minimizing the expectation value of the total Hamiltonian, $\langle\alpha|H|\alpha\rangle$ with respect to a variational mean field wave function for the quarks and a coherent state wave function for the soliton field subject to a constraint

$$\langle\alpha|Q|\alpha\rangle = Q_0(\alpha)\ , \tag{16}$$

where Q is some moment of the quark distribution

$$Q = \int\bar{\psi}\ q(\vec{r})\psi\ d^3r\ . \tag{17}$$

The constrained mean field equations now assume the form

$$(\vec{\alpha}\cdot\vec{p} + \beta[g\sigma_0(\vec{r}) - \lambda q(r)] - \varepsilon_k)\ \psi_k = 0\ , \tag{18a}$$

$$-\nabla^2\sigma_0 + U'(\sigma_0) + g\sum_{k\text{-occ}}\bar{\psi}_k\psi_k = 0\ , \tag{18b}$$

where λ is a Lagrange multiplier. Instead of specifying the constraint function q(r) explicitly and solving the pair of equations (18 a&b), it is more physical to specify the function in square brackets

$$[g\sigma_0(\vec{r}) - \lambda q(r)] \equiv \mathcal{V}(\vec{r}) \;, \tag{19}$$

then solve for the $\psi_k(\vec{r})$, and then for $\sigma_0(r)$; there is <u>no</u> iteration involved. The self consistency is implicit. The constraint can now be "discovered" by solving

$$\lambda q = g\sigma_0 - \mathcal{V} . \tag{20}$$

We parameterize $\mathcal{V}(\vec{r}) = \mathcal{V}(\alpha,\vec{r})$ by a folding procedure. We consider two spheres of radius R with centers separated by a distance α, as shown in Fig. 3. For $\alpha > 0$, we define a function $\Theta(\vec{r})$ equal to unity if $\vec{r}$ lies in the interior of either sphere and zero otherwise. If $\alpha < 0$, we choose $\Theta(\vec{r})$ to equal unity only in the intersecting, lens-shaped volume; this gives a natural continuation of α from positive values (prolate shapes) to negative values (oblate shapes). The radii of the spheres are chosen so that the enclosed volume is independent of α, and equals $2\mathrm{x}(4/3)\,\pi R_0^3$, where R_0 is the radius of a three quark bag. (The constant volume approximation, which is valid for a relativistic Fermi gas, can be relaxed to, say, minimize the energy as a function of R for each α, or to let R be another shape parameter. α can assume all values, $-\infty < \alpha < +\infty$, and is equal to the separation of isolated bags for $\alpha > 2R_0$.

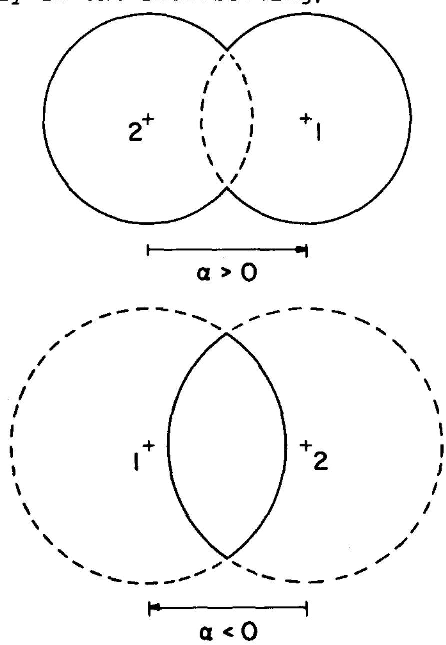

Fig. 3. Geometric shapes used to define $\Theta(\vec{r})$ and α.

Into this geometric shape is folded a Yukawa smoothing function, yielding

$$\mathcal{V}(\vec{r}) = g\sigma_V \, [1 - \frac{\mu^2}{4\pi} \int \frac{e^{-\mu|\vec{r}-\vec{r}\,'|}}{|\vec{r}-\vec{r}\,'|} \, \theta(\vec{r}\,') \, d^3\vec{r}\,'] \; . \tag{21}$$

Note that $\mathcal{V} \to g\sigma_V$ as $r \to \infty$ and $\mathcal{V} \cong 0$ for $\vec{r}$ well inside the geometric volume. It is adjusted to approximate the self consistent, unconstrained spherical solution for isolated bags.

The method of solution of equations (18 a&b) is a generalization of that described in ref. (5). Here, however, both $\mathcal{V}(\vec{r})$ and $\sigma(\vec{r})$ are expanded in terms of even Legendre polynomials, and the quark functions are expanded in terms of Dirac spinors of good quantum number κ. An example of the shape of $\mathcal{V}(\vec{r})$ is shown in Fig. 4. Low lying eigenvalues of the Dirac equation as a function of α are shown in Fig. 5. Parity is a good quantum number for the quark functions, and we note that for well-separated bags the eigenvalues become doubly degenerate with respect to the two parities, corresponding to degenerate left and right states. All of the GCM calculations described here were performed by A. Schuh, and the work is continuing.

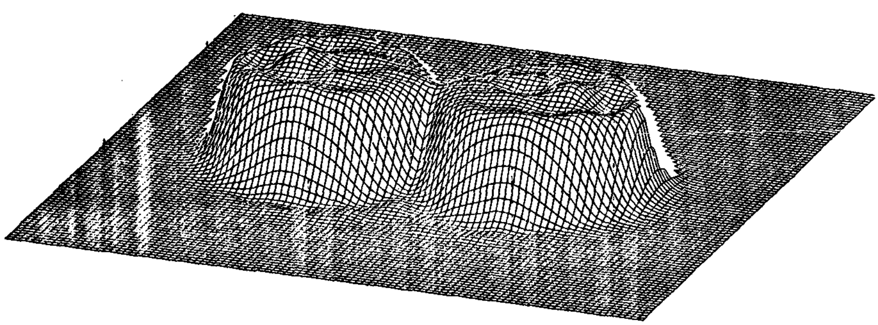

Fig. 4. The function $-\mathcal{V}(r)$ for α = 2 fm and R = 1 fm.

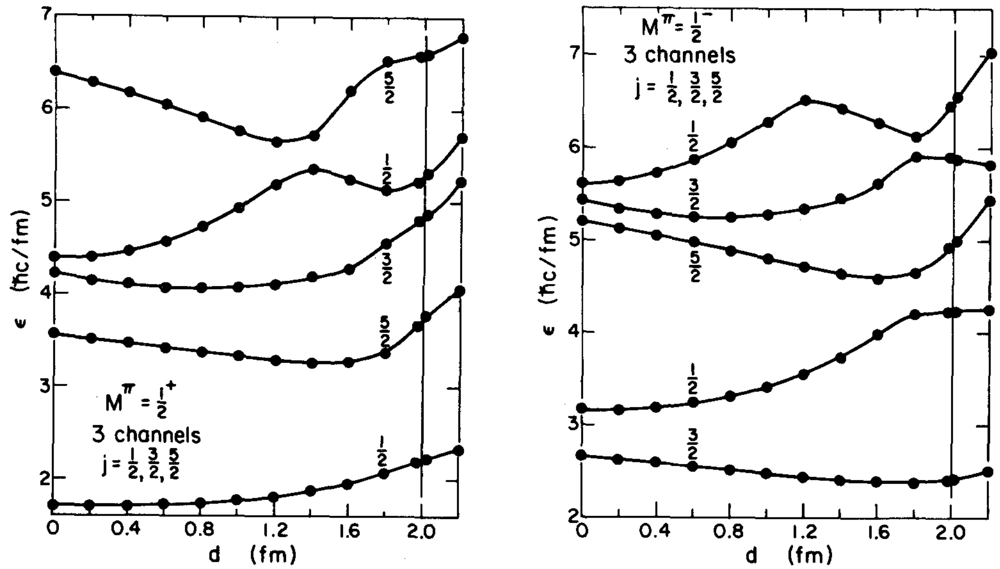

Fig. 5. Low lying quark eigenenergies calculated coupling three κ-states. Calculations coupling six states become flat, and exhibit degeneracy for + and - parities, for $\alpha > 2$ fm.

The GCM described to this point uses a static basis set, $|\alpha\rangle$. It is well known that for translational or rotational motion, where the GCM reduces to projection, the method fails to reproduce the proper mass or moment of inertia. The rectification of this problem in those cases has been addressed by many authors. We follow the self-consistent cranking method of Haff and Wilets[14], which builds dynamics into the wave function. In brief, we replace

$$|\alpha\rangle \rightarrow e^{i\beta(Q-\langle Q\rangle)}|\alpha\rangle \tag{22}$$

where β can be identified with the momentum conjugate to $\langle\alpha|\,Q\,|\alpha\rangle = Q_0(\alpha)$ in the resulting differential equation for $\phi(\alpha)$; that is,

$$\beta = -i\ \partial/\partial\langle Q\rangle = -i(dQ_0/d\alpha)^{-1}\ \partial/\partial\alpha. \tag{23}$$

In the translational case, where $Q = R_{GM}$(the center-of mass operator), $Q_\alpha = \langle Q\rangle = Z$ (the center of the bag), $\beta = P$ (the total momentum), $\phi = e^{iPZ}$, then the result is completely equivalent to projection into the zero momentum state followed by boosting to a state of some finite momentum:

$$e^{iPR_{cm}} \int_{-\infty}^{\infty} dZ \, |Z\rangle.$$

Further calculations are in progress, of which we mention a few: the structure of the virtual mesonic cloud around the nucleon; further corrections to hadronic spectra and the evalaluation of decay widths; transition from nucleonic matter to the quark-gluon plasma as function of density and temperature, using periodic (lattice) boundary conditions for the soliton field.

We have not begun, but would recommend to others who have the machinery, to do time-dependent mean field (TDMF) calculations, analogous to time-dependent relativistic Hartree-Fock calculations.

We conclude by re-emphasizing the utility of the model in calculating dynamical processes. Reactions which involve collision, fission, fusion, creation and oscillation of bags can be and are being calculated using the well-developed techniques of nuclear collective models. Such processes are difficult, at best, to calculate in other relativistic bag models.

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