EFFECTIVE QCD-LAGRANGIAN FOR NUCLEAR PHYSICS

E.R.Hilf and L.Polley
Institut fuer Kernphysik, T.H.Darmstadt, D-6loo Darmstadt, FRG

SUMMARY

A semi-classical approach to Nuclear Physics is proposed, which makes use of the one-loop effective thermal action for the Yang-Mills field, and the c-number colour source approximation to quarks.

NUCLEI AS COMPLEX SYSTEMS

Atomic Nuclei are complex systems which can be either disintegrated and fragmented into free nucleons,-or, in principle, be crunched to a plasma,- which both are objects, formed by the interaction of quarks by virtue of the strong interaction mediated by the gluon field $A_\mu^a(x)$.

Nuclei show some similarities to Solid Grains which also can either be disintegrated and fragmented into free molecules, or be crunched to a Plasma. Both are complex objects, formed by the interaction of charged Nuclei and Electrons, by virtue of the electro- magnetic interaction mediated by the photon field $A_\mu(x)$.

Theoretically the exact theory to treat a Solid Grain is known, the QED(or at least the QM). With it one may try to calculate and thus understand the properties of the individual free molecules.

On the other hand classical limits of the theory may be sufficient to treat the ionic plasma. However it is hopeless to try to calculate the properties of the complex system, the Grain, by the full theory. But it is also not necessary. As is wellknown it is sufficient to use semiclassical concepts derived from the exact theory such as the WKB-method, or the selfconsistent Thomas-Fermi. These methods which contain no additional parameters are well suited to study the slight disturbance of the outer part of the atomic Electron shell, the atomic polarisation, which results in the saturation binding,- covering a very small but important sector of the average density.

Analogously, the nuclear structure may ultimately be successfully described in terms of semiclassical concepts, derived from QCD, which contain no additional parameters, whereas it is hopeless to treat such extended systems (the Pb^{208} nucleus covers about $10^3 fm^3$, even a single nucleon about $6 fm^3$) with full QCD, say on a lattice, due to the semiinfinite necessary computing time (sum over all links at present possible for fractions of a fm^3 only).

The wellstudied bag-type models although calculable, contain parameters fitted to free hadrons and thus are open to some arbitrariness in their semiempiric describing the polarisation of nucleons in nuclei. The soliton model does offer an however (parametrised but consistent) simplified scalar microscopic field description.

The to be developed Semiclassical Concepts may be useful for treating other complex systems composed of complex QCD-objects, such as Glue-Crystals of Glueballs, pion condensates of pions, neutron- and hadron-star matter.

Some experimental facts on nuclei to be understood by a semiclassical concept are the increased formfactor of nucleons in nuclei (see Rith, this volume), and the nucleon-nucleon force in nuclei. The latter can be formulated in terms of the semidivergence of the action-expansion, which is then normally stated in terms of a (fitted) density-dependence of the coefficients of the Hartree-Fock (or extended Thomas-Fermi)Hamiltonian. Fit data are experimental data[1] for the nuclear ground state such as the saturation density $\rho_o = (4\pi r_o^3)^{-1}$ with r_o=1.17fm, the saturation energy density of about 16MeV, the compressibility $K_o \approx 250$MeV, the surface tension

$a_s \approx 20$MeV, the asymmetry energy of 32MeV, and on the other hand e.g. recent experimental data[2,3] for warm nuclei, yielding a critical temperature of $T_c = 12$MeV and a critical exponent of $\kappa = 1.8$, which are not in contradiction to some density-dependent Hartree-Fock scheme[4], but different to what one had expected from an earlier simple quadratic velocity dependence of a selfconsistent Thomas-Fermi approach[5]. A full-scale evaluation of all these nuclear informations will be the topic of a future conference[6].

THE SEMICLASSICAL METHOD

Wishes to a semiclassical approximation of QCD are that it should reproduce asymptotic freedom, and confinement.

The quarks, assumed to be confined in the nucleons, even if their distribution might be slightly polarised if embedded in a nucleus, are dealt with by treating them as external c-number sources $j_\mu^a(x)$, and we will discuss in this paper only static sources.

The gluons are represented by the expectation values of their field operators,

$$A_\mu^a(x) = \langle \hat{A}_\mu^a(x) \rangle \ .$$

The system is assumed here to be in its groundstate, thus we look for the limit $T \to 0$. But still there are contributions from quantum fluctuations of the field.

Then we can make use of the thermodynamic method[7,8]. For a given Hamilton operator as a functional of the classical sources H[j] the partition function is given by

$$Z[j] = \text{tr} \exp(-\beta H[j]) \ . \tag{1}$$

This partition function is non-zero, for we do not impose unphysical boundary condition. as in some early lattice calculations, where periodic b.c.

were imposed, resulting as could be proved[9] in an empty Hilbert-space and a trivial partition function. If then formally Z=0 was assumed and inserted into (2), then the numerical results were interpreted as a first order phase transition, although this just reflects the fact[10] that the screening length exceeds the chosen lattice size.

For nonzero Z the Free Energy is

$$F[j] = -(1/\beta)\log Z[j] \tag{2}$$

From this thermodynamic potential we obtain the new effective action, supposed to be dependent on the conjugate variable of $j^a_\mu(x)$,

$$A^a_\mu(x) = -\delta F[j]/\delta j^{a\mu}(x) \tag{3}$$

by the contact-transformation

$$\Gamma[A] = F[j[A]] + \int d^3x\ j^{a\mu} A^a_\mu\ . \tag{4}$$

The functional derivative of this action yields the field equations

$$\delta\Gamma[A]/\delta A^a_\mu(x) = j^{a\mu}(x) \tag{5}$$

its local density is the effective Lagrangian L[A],

$$\Gamma[A] = \int d^3x\ L_{eff}[A(x)] \quad . \tag{6}$$

In alliteration to Classical Mechanics terms (5) is the Euler-Lagrange equation. Only formal manipulations have been involved so far, exact in principle.

In the past a somewhat different approach was proposed[7]. From L[A] a local classical Hamiltonian was gained by a contact-transformation,

$$H[j,A] = jA - L[A]$$

yielding a global function

$$F[j,A] \doteq \int d^3x\ H[j,A],$$

which is not a thermodynamic potential, in contrast to (2). Still its functional derivative

$$\delta F[j,A]/\delta A|_{j=const} = j$$

is one of the equations of motion of the Hamilton-Jacobi formalism analogue. For static sources it coincides with (3). Sometimes $-H[j,A]$ was called a Lagrangian[11], which was tried to be cured by a new variational principle, but yielding a spurious sign, which disagrees with the respective QED limit.

THE EFFECTIVE LAGRANGIAN

Before introducing the effective gluonic Lagrangian we remind of the photonic analogue.

The Hamiltonian for photons and external electromagnetic sources j, is known,

$$H[j]=\sum_{\lambda=1}^{2} \int d^3k\ \ a^*(k,\lambda)\ \omega a(k,\lambda)+\sum_{\lambda=1}^{2}\ \int d^3k/\sqrt{(2\omega)}\ \times(j(k,\lambda)a^*(k,\lambda)+h.c.)$$
$$+ \int\int d^3x d^3x'\ \ \rho(x)\rho(x')/(8\pi|x-x'|)\ . \quad (7)$$

so the partition function and the Free Energy can be calculated. The partition function factorises into a free photon part and a source-dependent part. In the zero temperature limit, only the latter contributes to the Free Energy,

$$F[j]= \int\int d^3x d^3x'(\rho\rho'-jj')\ /(8\pi|x-x'|)\ . \quad (8)$$

Executing the thermodynamic method (section 2) yields

$$L_{eff} = \tfrac{1}{2}(B^2-E^2) = F_{\mu\nu}F^{\mu\nu} = F^2 \tag{9}$$

This coincides with the classical Lagrangian of the electromagnetic field.

In classical Yang-Mills Theory, the purely classical action is of similar form,

$$L = g^{-2}F^2$$

where

$$F^a{}_{\mu\nu} = \delta^{ac}(\partial_\mu A^c_\nu - \partial_\nu A^c_\mu) + f^{abc} A^b_\mu A^c_\nu . \tag{10}$$

The field equations are

$$(\delta^{ac}\partial^\mu + f^{abc}A^{b\mu})\, F^c_{\mu\nu} = j^a_\nu , \quad \nu=0,1,2,3. \tag{10a}$$

The field equations have been solved numerically for different classical colour-charge configurations, such as the one charge shell[12], two charges[13], periodic plates[14]. Neither confinement nor asymptotic freedom was found.

One loop constant background-field calculations[15], as well as renormalisation group arguments[16], have suggested to consider an effective action that is obtained by replacing the bare coupling constant of the classical action with a running coupling which depends on the local colour field strength in units of a renormalisation group invariant mass.

This approach may also be plausible in that the running coupling constant of perturbative QCD has a relative momentum q^2 under the logarithm which in a thermodynamic framework would be proportional to the temperature or to F^2.

The effective QCD-Lagrangian is obtained by inserting the one-loop running coupling constant into the Lagrangian, yielding

$$L_{eff} = (1/4g^2)\ F^a_{\mu\nu} F^{a\mu\nu} \log[\tfrac{1}{2}F^a_{\ \mu\nu} F^{a\mu\nu}/e\kappa^2]. \tag{11}$$

The properties of this ansatz will be evaluated in section 4.

A quite different approach[18], starting from the Dyson equations in the limit $q_o=0$, $q\to 0$, resulted in $L=FD^2F/\mu^2$.

It seems that the discrepancy between this and the effective action used in this paper is due to different choices of renormalisation conditions[19]. Roughly speaking, the running coupling of Baker[18], $\bar{g}^{-2}=D^2/\mu^2$, coincides with the bare coupling at a particular momentum $D^2 = p^2 = \mu^2$, whereas the running coupling of refs. 13,14 does so for a particular value of the field strengths, $F^2 = \mu^2$.

PROPERTIES OF THE EFFECTIVE LAGRANGIAN

Qualitatively, L_{eff} is displayed as a function of B^2-E^2 in fig.1. Its prominent feature is the local minimum at $B^2-E^2 = \kappa^2$, which is even an absolute minimum for configurations with a real action (complex action configurations which have $B^2<E^2$ somewhere, are regarded as unstable.)[16].

For vacuum configurations, i.e. if j=0, the free energy equals the action by (4), and also the energy for T=0. Thus a thermodynamically stable zero temperature vacuum solution has to fulfill the extremum condition

$$\delta\Gamma/\delta A = 0 \quad \text{or} \quad dL/dF^2 = 0 \ . \tag{12}$$

This indeed is an Energy minimum and thus thermodynamically stable. Because this minimum for L of the vacuum occurs at

$$B^2 - E^2 = \kappa^2 > 0 \ . \tag{13}$$

the stable vacuum has nonvanishing colour-magnetic fields which are stronger than the electric fields.

The dielectric function

$$\varepsilon = dL/dF^2 = \log F^2/\kappa^2. \tag{14}$$

is displayed in fig.2. For the vacuum we have $\varepsilon=0$.

The effective Lagrangian, although developed from the one-loop approximation, is proposed and taken to be a useful approximation for all configurations of static sources.

The reason is, that for the vacuum $\varepsilon=0$, as to be expected[15], while with larger $|\varepsilon|$, signalling the presence of colour-charges, the running coupling $\bar{g}^2 = 1/\log(F^2/\kappa^2)$ goes to zero.

For saturation properties of nuclear matter, where we are not interested to study the deep interior of the nucleons, but their slight gluon-cloud polarisation due to the presence of neighbouring nucleons, the application of our pure gluonic L_{eff} seems to be reasonable.

Although qualitatively we do expect, because of the correct limits of either ε or g^2, that the L_{eff} will describe confinement, we have to study explicitely how this comes about.

CONFINEMENT WITH L_{eff}

Confinement in this rather simplified approach should mean that for a given charge-distribution, restricted to a finite volume in space, the expectation values of the Gluon fields should asymptotically, that is sufficiently far from the sources, tend to fulfill the vacuum condition $B^2-E^2=\kappa^2$, and that at least solutions for non-zero total charges Q should energetically be infinitely unfavourable. We shall see, that in our context it will be nontrivial to find solutions for the fields at all, and that we were successfull only in SU_{3c}, and that it is possible only if the vacuum condition holds identically outside a finite volume, which contains a colour charge distribution of total colour charge zero. Thus L_{eff} seems to enforce a rather strict version of confinement.

To obtain some quantitative results by analytical considerations we proceed by assuming a reasonable function $\varepsilon(r)$, namely $\varepsilon \approx 1$ inside the colour-charge distribution and $\varepsilon \approx 0$ far from sources. Next we try to find a set of solutions to the Yang-Mills analogue of the Biot-Savart law (spatial components of Maxwell's equations) for zero current density. Then we pick the solution which fullfills $\varepsilon = \log F^2/\kappa^2$, and if there is one, we can calculate the actual colour-charge distribution from $\rho = D_i \varepsilon F_{io}$.

Let us start with an Abelian exercise. With only one relevant colour direction assumed, all selfcoupling terms vanish, yielding $\nabla(\varepsilon E) = \rho$, $\nabla x(eB) = 0$ with $E = \nabla A_o$ and $B = \nabla x A$.

For the vacuum abelian solutions with $B^2 \geq \kappa^2$ do exist with $\varepsilon = 0$ and $\rho = 0$ everywhere.

Let us prescribe $\rho \neq 0$ in a small region. Then in the vicinity $\varepsilon \neq 0$, but $\varepsilon \approx 0$ far away from the source region. Let us assume for simplicity $\varepsilon \equiv 0$ outside a surface $S = \partial V$ more general proof with $\varepsilon \approx \exp(-\lambda r^2)$ yields the same result). Close to S but inside, the smooth function ε will be small and the field equation yields $\nabla x B + B x \nabla \ln\varepsilon = 0$ with the second term getting very large near S, while $\nabla x B$ is assumed to be continuous. Thus at S we have B to be parallel to $\nabla \ln\varepsilon$, that is parallel to the surface normal n, and $B^2 \geq \kappa^2$ because $\varepsilon = 0$. Since the field configuration would thus form a colour magnetic monopole in contradiction to the starting point $B = \nabla x A$, the result is that in this simple Abelian exercise no solutions exist,- the vacuum cannot be perturbed by introducing localised charges.

To obtain a simple nonabelian field configuration we consider spherically symmetric fields only. The spherically symmetric ansatz by Wu and Yang[20], which is a simple special case of Witten's ansatz, is applicable for our semi-classical model too. For the SU_{2c} gauge subgroup, it reads

$$A_o^a = r^{-1}\, G(r) r_a \quad , \quad A_i^a = r^{-2}(H(r)-1)\varepsilon_{ian} r_n \quad . \tag{15}$$

$$\rho^a(r) = r^a \rho(r)/r^3 \tag{16}$$

with the boundary conditions

$$H(0) = 1, \; H'(0) = 0, \; G(0) = 0 \; , \tag{17}$$

which yields

$$B_i^a B_i^a = 2(H')^2/r^2 + (H^2-1)^2/r^4 \tag{18}$$

$$E_i^a E_i^a = (G')^2 + 2G^2H^2/r^2 \quad , \tag{19}$$

so that the equations of motion take the form of two coupled ordinary differential equations of second order

$$-H'(\ln|\varepsilon|)' = (H''+H(1-H^2)/r^2+G^2H) \tag{20}$$

$$\rho(r) = (r^2\varepsilon G'(r))' - \varepsilon GH^2 \text{ with } \varepsilon = \log F^2/\kappa^4. \tag{21}$$

For 'soft' bags, indicated by ε going smoothly to zero as $r \to \infty$, but with $\varepsilon \neq 0$ everywhere, it has been proven[17] that there are no continuous smooth solutions for the fields, since $H(r)$ runs into a singularity at some finite r.

For 'hard' bags, with $\varepsilon \equiv 0$ beyond some small r_1 outside V, a somewhat lengthy argument[17] dealing with inequalities leads to $H'(r_o) \equiv 0$ as a further boundary condition, and to $H(r_o)^2 < 1$.

Then there exist solutions of the sourceless field equation $D_\mu \varepsilon F_{\mu k} = 0$ if

$$r_1 \leq r_o \leq 1/\sqrt{\kappa}, \tag{22}$$

which with the adopted values of κ of 2oo MeV is about $r_o \leq 1$fm.

In this context, let us say, a proton would be described as a classical colour-charge distribution of, say, o.8fm radius, surrounded by a polarised gluon cloud, which at r_o slides into a gluon-vacuum solution with $B^2 - E^2 = \kappa^2$.

A special solution of $D\varepsilon F=0$ can be gained by choosing $-1\leq H(r_o)\leq 0$ and $G(r)\equiv 0$, then the ansatz $H(r)\approx H_1(r)=[1-\kappa(1+\delta)r^2]^{\frac{1}{2}}$ for $r \leq r_1 << r_o$, $H(r)\approx H_2(r)=H(r_o)+\alpha(r-r_o)^3$. for $r\leq r_o$ with some smooth interpolation yields a solution with the features

$$|\varepsilon| \approx \exp(-c/(r-r_o)) \quad \text{for } r<r_o, \tag{23}$$

$$B^2(r)\geq B^2(r_o)\geq \mu^4(1+\delta)^2. \tag{24}$$

These solutions do not yet satisfy the consistency requirement that $\varepsilon(r)=\log F^2(r)/\kappa^2$.

In a larger gauge group, SU(3), the 8-component of the charge density and the colour electric field can be used to adjust the total magnitude of the field strength to the value required by the dielectric function $\varepsilon(r)$ fixed in advance. We only obtain, as an additional equation,

$$\nabla\ (\varepsilon E^8) = \rho^8 \tag{25}$$

which is easy to solve.

The total colour charge is the integral of $\mathrm{div}(\varepsilon E)$ over the bag volume, which can be evaluated as a surface integral in a region where ε vanishes identically. Thus for the Wu-Yang radial symmetric SU_2 fields plus an ρ^8 source a bag-type colour-singlet solution is found and constructed.

Of course, $r_o=1/\sqrt{\kappa}$ is not to be compared with the bag-radius of bag models. Instead this bag-radius would be somehow defined by the distribution of the colour charge distributions. Then at $r_B\leq r_o$ the undisturbed gluon vacuum is reached.

L_{eff} AND NUCLEAR PHYSICS

A first consequence of the results would be that the interaction of free nucleons vanish at latest when their half-distance is larger than $1/\sqrt{\kappa}$.

Comparing this to the average nucleon half-distance[1] inside nuclei of about 1.17±0.01 fm, we see that in this frame the wellknown nuclear saturation density sets an upper limit for the renormalisation group invariant mass, for

$$1.17 \approx r_{n.m.} < r_o \approx 1/\sqrt{\kappa}. \tag{26}$$

Thus $\sqrt{\kappa}<169$ MeV.

Since $\sqrt{\kappa}$ is defined using field strengths as a renormalisation point, it cannot directly be compared to the more widely used invariant mass Λ, defined by using a momentum space renormalisation point. However, an estimate[21] within the perturbation limit, using the bag model estimate for the running coupling constant $\alpha_c(p_f)\approx 2.2$ yields for the known nuclear matter density $n \approx 0.15$ fm^{-3} the value of $\Lambda\approx 107$ MeV. If this value would be inserted into (28) a zero force nucleon-nucleon half-distance of 1.82 fm would result which is not in contradiction to free n-n scattering data.

Numerical studies of solutions starting from L_{eff} for various given external source configurations are under way.

For application to the interior configurations of strongly interacting systems in equilibrium such as a free nucleon first the approach has to be improved by treating the quarks as Dirac particles, with triplet colour charges.

Finally, for the interior of quark systems, the colour-dielectric function ε may depend not only on the field strength F^2 but also on other invariant quantities.

The complex spatial geometry of multinucleon systems such as the nucleus may,- for studying the polarised nucleon formfactor, the density dependence of the nucleon force etc.,- well be approximated by studying the same problem of a nucleon in a nuclear surrounding but in the simpler spherical geometry.

This method is wellknown in Solid State Physics, named after Wigner and Seitz, where the calculation of the electron cloud of an atom in a spherical cell, and with the boundary conditions of zero Coulomb potential gradient at the surface and total charge zero yields the binding energy per atom of solids to better than 2%. A similar approach was used for nuclei in a thin neutron gas occuring in the neutron star skin[22]. Thus already in solids the gain in binding due to specific crystalline configurations but with the same total density is small. For nucleons in nuclei this approximation should be even better due to that the shell effects for (heavy) nuclear masses are only of order of less than 10 MeV[1] which is e.g. for Pb^{208} only 0.6%, reflecting the large ratio of kinetic to binding energy per nucleon.

At present, this method is applied in various approaches to semi- classical QCD.

In the Soliton Model[23], treating the nucleon as three Dirac- particles coupled to a colour-singlet scalar field σ, the soliton field, the selfconsistent solutions for ψ_q and $\sigma_o(r)$ are known. Thus for the application to a nucleon within a nucleus only the boundary conditions for the σ-field have to be changed to

$$\sigma'(r_1) = 0 \; , \qquad (27)$$

where r_1 is given by the nuclear density

$$\rho_o = 1/(4\pi r_1{}^3/3). \qquad (28)$$

Because $\sigma(r)$ will be only slightly different from $\sigma_o(r)$ in the region where the quark-density is non-zero, the soliton model equations can be solved[24] perturbatively to yield selfconsistent solutions for the slightly polarised quark density and the slight binding increase.

In the effective Lagrangian approach, the respective boundary condition is the vanishing of the radial derivatives of $A^a_\mu(x)$ at r_1. However for the respective minimisation either an additional constraint is needed, say,the square of the total charge of the external colour-charge density to be kept constant, or better the concept is to be improved by quantising the sources. Work in that respect is in progress.

ACKNOWLEDGEMENTS

One of us (E.H.) gratefully acknowledges stimulating discussions and the hospitality of the Nuclear Theory Division, University of Washington, Seattle, during the Summer Institute of Theoretical Physics, the attendence of which was supported by the Gesellschaft fuer Schwerionenforschung.

REFERENCES

[1] H.v.Groote,E.R.Hilf,and K.Takahashi, Atomic Data and Nuclear Data Tables 17(1976)418
[2] X. Panagiotou,D.Scott et al.Preprint MSUCL,Sept.(1983)
[3] D.Scott, preprint MSUCL(1983)434
[4] G.Sauer,H.Chandra,U.Mosel, Nucl.Phys.A264(1976)221
[5] W.A.Kueppers,G.Wegmann,E.R.Hilf,Ann.of Physics,88(1974)454
[6] Int.Conf.Atomic Masses and Fundamental Constants, VII, Bad Seeheim, FRG; September (1984)
[7] S.L.Adler, Phys.Rev.D23(1981)2905
[8] S.Coleman and E.Weinberg, Phys.Rev.D7(1973)1888
[9] E.R.Hilf and L.Polley, Phys.Lett.(1983) in press;
[10] B.Svetitsky,priv.comm.(1983)
[11] S.L.Adler and T.Piran, preprint(1983)and refs.therein
[12] M.Magg,Nucl.Phys.B158(1979)154
[13] R.Freedman,L.Wilets,S.D.Ellis,E.M.Henley,Phys.Rev.D22 (1980)3128
[14] L.Carson, and E.R.Hilf, unpublished(1980);
[15] S.K.Savvidy, Phys.Lett.71B(1977)133; H.Leutwyler, Nucl.Phys.B179(1981)129
[16] H.Pagels and E.Tomboulis, Nucl.Phys.B143(1978)485
[17] E.R.Hilf and L.Polley,(1982), submitted to Nucl.Phys.
[18] M.Baker and F.Zachariasen,Phys.Lett.108B(1982)206 and preprint40048-16 P3 May(1983)Univ.of Washington
[19] M.Baker and D.Pottinger, priv. comm.
[20] For a review, see A.Actor, Rev.Mod.Phys.51(1979)461
[21] G.Baym, in"Proc.Statist.Mech.of Quarks and Hadrons", ed.H.Satz, North-Holland Publ.Comp.(1981)461
[22] E.R.Hilf,'Early Neutron Star Matter and Hadrons', in "Int.Conf.Mesonic Effects in Nuclear Structure, Wiss.Verlag Bibl.Inst.(1975)17 and refs. therein;
[23] L.Wilets, this volume;
[24] A.Schuh,L.Wilets,and E.R.Hilf, in preparation

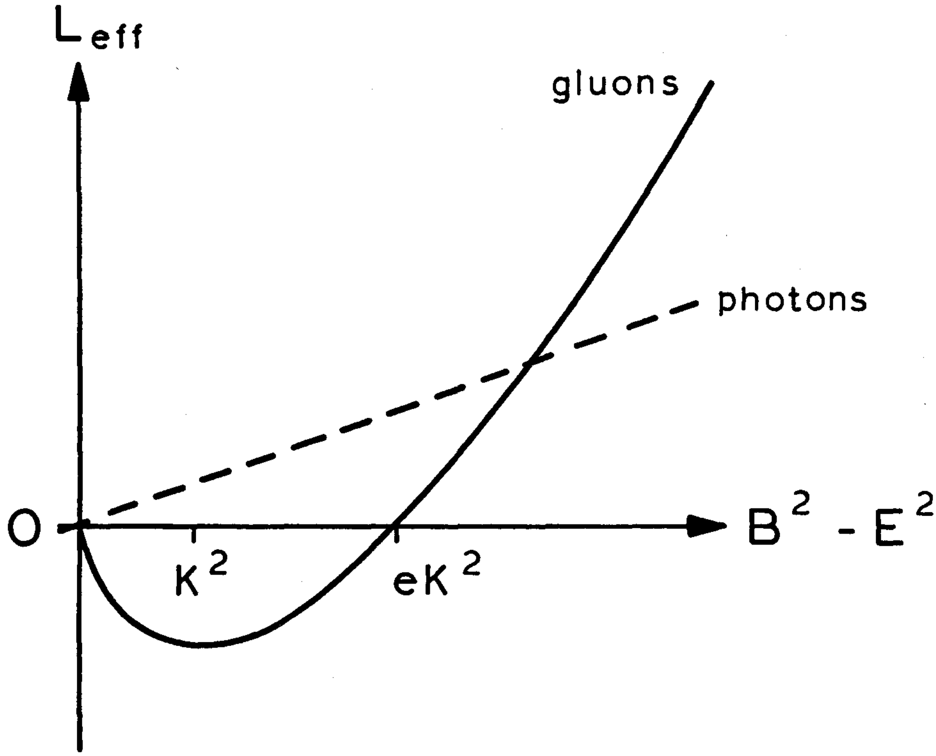

Fig.1

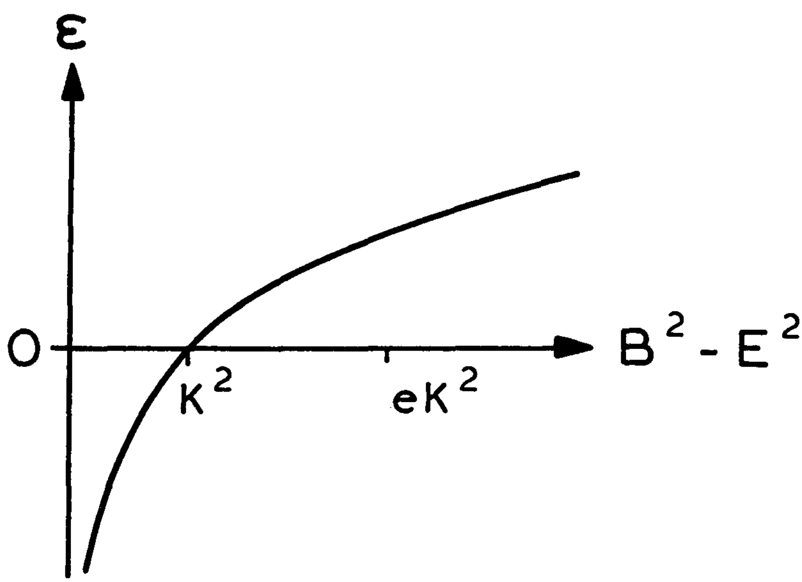

Fig.2