QUANTITATIVE SPECTROSCOPY WITH QUARKS

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We discuss heavy quarkonium spectroscopy in potential models, as well as the extension of these potential models to hadrons containing ordinary quarks.

In this talk, I shall not speak at all of nuclear structure in terms of quarks, but only of the description of "elementary" particles in terms of quarks. Initially, the quark model was largely a mnemonic trick to remember which states are allowed and which are forbidden. In the eyes of Murray Gell-Mann, it was a procedure to write currents with the right commutation relations. It is only in 1974, when S. Ting and B. Richter discovered the J/ψ and when people understood that this very narrow particle (60 keV for a mass of 3 GeV !) could not be something else than a heavy quark-antiquark bound state, that things changed At that moment it was understood that quarks, or, at least heavy quarks like the c quark predicted by S. Glashow-J.Iliopoulos-L. Maiani in 1970, behaved very much like real particles, except for the fact that maybe they cannot be isolated. This was in fact from the very beginning the point of view of George Zweig.

It was natural to describe the $c\bar{c}$ system, of which two levels were known from the very beginning, the J/ψ (mass 3095 MeV) and the ψ' (mass 3685 MeV) as a system of two heavy objects interacting by a potential and obeying the Schrödinger equation. However, unlike what happens in the hydrogen atom, the quark-antiquark potential was not known. However, the theoretical prejudice is that one gluon exchange dominates at short distances while at large distances a linear rising confining potential seems to be the most natural thing to prevent quarks from escaping. So the initial potential used was

$$V = -\frac{4}{3}\frac{\alpha_s}{r} + Cr .$$

The potential model description was tremendously successful in a qualitative and semi-quantitative way:

- the existence of P states between the ψ and ψ' was predicted. A triplet of P states was discovered. The non-observation of the singlet P state is due to the difficulty of producing it.

- A D state, with the same quantum number as the photon, was predicted with an energy slightly higher than the ψ' and was observed in e^+e^- collisions.
- Radial excitations of the S states were predicted and seen.
- Pseudoscalar states η_c and η_c' respectively associated with the vector states $J\psi$ and ψ' were predicted and, after a complicated history, observed experimentally.

In 1977 another heavy quark system was found, the T, made of a $b\bar{b}$ pair, as indicated by an indirect measurement of the absolute value of the charge of the constituent quarks. The T, with a mass 9459.9 ± ± 0.1 MeV (notice the fantastic precision!) is much heavier than the $J\psi$ and logically the potential description should work still better. Now, in fitting the energy levels of the T, we have the extra constraint that the potential should be approximately the same. This is "flavour independence": because QCD is a non-Abelian gauge theory, the quark-quark-gluon coupling is related to the three gluon coupling and, therefore, cannot depend on the particular flavour of the quarks. It is not completely obvious that the resulting potential (if such a notion makes sense) will be exactly flavour independent, but the deviations from flavour independence should go to zero when the mass of the quarks goes to infinity. However, we can try to assume "precocious flavour independence" and try to impose that the $b\bar{b}$ and $c\bar{c}$ potentials be exactly the same.

It is at this point that we enter into the domain of quantitative spectroscopy. The question is what is the quark-antiquark potential. In principle, the quark-antiquark potential could be obtained from lattice gauge theory calculations, and this is on the way. However, lattice gauge theory would have a hard time getting the short distance behaviour, at distances less than the lattice spacing. However, the short distance behaviour can be obtained from asymptotic freedom. This was done first by H. Krasemann and S. Ono[1], J. Richardson[2] and W. Buchmüller, G. Grunberg and S. Tye[3]. One has at short distances:

$$\left.\begin{aligned} V &= -\frac{4}{3}\frac{\alpha_s(r)}{r} \\ \alpha_s(r) &= \frac{12\pi}{25\log\left(\frac{1}{r^2\Lambda_{\overline{MS}}^2}\right)} \end{aligned}\right\} \qquad (1)$$

Notice that this attenuation of the Coulomb singularity is already not negligible for the T spectrum but becomes crucial for the $t\bar{t}$ system, since we know that the mass of the t quark is superior to 20 GeV.

At larger distances, while H. Krasemann and S. Ono just connect the potential (1) to the linear confining potential with a logarithmic part, as was done before by G. Bhanot and S. Rudaz[4)], J. Richardson and W. Buchmüller (whose more refined short range potential contains two loop effects) have a nice analytic interpolating formula in momentum space.

In contrast with these QCD inspired approaches, one can also take a purely phenomenological point of view and try to find the simplest potential fitting, the $c\bar{c}$ and $b\bar{b}$ spectrum. Historically, the first attempts in this direction were made by Y. Tomozawa, C. Quigg and J.L. Rosner. The latter pointed out that if the spacings between <u>all</u> levels of the $c\bar{c}$ system were the same as the spacings between the levels of the $b\bar{b}$ system, this would be a proof that the potential is exactly $V = C \log(r/r_0^*)$. It is a simple exercise to prove, rescaling r, that if $V = \log r$, the distances between the levels do not depend on the mass of the quarks. The converse is more difficult to prove. In any case, as one can see from Table 1 (which is much more complete now!) this property is not very far from being satisfied by the $c\bar{c}$ and $b\bar{b}$ spectrum. We notice, however, a tendency of the $b\bar{b}$ levels to have a somewhat smaller spacing. To take this into account, we can consider the simplest generalization of the log r potential, $V = A + Br^{\alpha}$, with α slightly positive. This is what I tried[5)], taking

$$V = -8.064 + 6.870\, r^{0.1} \tag{2}$$

where the units are GeV for energies and GeV^{-1} for distances.

In Table 1, we give the level spacings, from the 1^{--} ground state, to the various L = 0, L = 1 and L = 2 states (for L = 1, we give the centre of gravity of the triplet states), and the relative leptonic width of the L = 0 state, $\Gamma_{ee}(nS)/\Gamma_{ee}(1S)$ calculated "naively" using the formula $\Gamma_{ee} = C|\psi(0)|^2/M^2$. We give the experimental numbers[6)], the theoretical predictions of the Buchmüller potential and the predictions of potential (2). The absolute masses of the various states are not significant since the ground state is adjusted by fixing the effective quark mass.

	$c\bar{c}$			$b\bar{b}$		
1^{--} ground state	3095 MeV			9460 MeV		
Quark mass	Exp.	Buchmüller 1480 MeV	Martin 1800MeV	Exp.	Buchmüller 4870 MeV	Martin 5174 MeV
2S - 1S $\Gamma_{ee}(2S)/\Gamma(1S)$	589 0.46 ±0.06	600 0.45	592 0.40	561 0.43	560 0.44	560 0.41
3S - 1S $\Gamma_{ee}(3S)/\Gamma(1S)$	935 0.16?	1020 0.32	937 0.25	890 0.32	890 0.32	900 0.35
4S - 1S $\Gamma_{ee}4S/\Gamma(1S)$	1319?	1380	1185	1116 0.21	1160 0.26	1140 0.27
1P - 1S	425	420	407	440	430	400
2P - 1S				796	790	780
1D	677	710	692			

Table 1: Excitation energies and relative leptonic widths for the $c\bar{c}$ and $b\bar{b}$ systems compared with the theoretical predictions of Buchmüller et al. and Martin.

Table 1 does not incorporate the data on the pseudoscalar states η_c and η_c' whose spacing to the corresponding vector states are

$$J\psi - \eta_c = 112 \text{ MeV}, \qquad \psi' - \eta_c' = 93 \text{ MeV},$$

for which W. Buchmüller et al.[3], as well as previously D. Beavis et al.[7] have predictions of the right order of magnitude but, however, too low by a factor 0.8.

There are many other fits of the data but I have chosen these two as representatives of "theoretical" and "phenomenological" potentials. In both cases, the agreement with experiment is strikingly good. The flavour independence of the potentials is perfect (in fact too perfect maybe!). In the phenomenological potential the number of parameters is only three. If we look closer at the figures, we see that the phenomenological potential is somewhat better for higher L = 0 excitations. This may, however, not be terribly significant since open and closed coupled channel effects are present (remember that the $2m_D = 2 \times 1865$ MeV = 3730 MeV and $2m_B = 2 \times 5274 = 10548$ MeV). What matters more is that the QCD motivated potential reproduces much better the P states, especially the 1P state, than the phenomenological potential. This is presumably due to the fact that a non-singular potential overestimates the mass of the L = 0 states; this, however, is corrected by adjustment of the quark mass and leads to underestimating the mass of the L = 1 states.

In the most ambitious attempts, one tries also to reproduce the separation between the P states due to spin dependent forces. This is especially interesting now that we have two triplets of P states. Generally, the spin forces appear from a reduction of the Breit equation, but the nature of the interaction in the Breit equation is model dependent. We give in Table 2 some theoretical predictions by D. Beavis et al.[7], W. Buchmüller[8] and E. Eichten and F.L. Feinberg[9]

		Exp.	Buchmüller	Eichten Feinberg	Beavis
$c\bar{c}$	$2^{++}-1^{++}$	44 MeV	57 MeV		69 MeV
	$1^{++}-0^{++}$	93 MeV	83 MeV		71 MeV
$b\bar{b}$	$2^{++}-1^{++}$	20 MeV	19 MeV	26 MeV	23 MeV
χ_b	$1^{++}-0^{++}$	21 MeV	28 MeV	25 MeV	24 MeV
	$2^{++}-1^{++}$	16 MeV	18 MeV	17 MeV	17 MeV
χ_b'	$1^{++}-0^{++}$	21 MeV	21 MeV	17 MeV	24 MeV

We see that orders of magnitude are reasonable but agreement is not perfect in any of the three calculations. For $c\bar{c}$, QCD sum rules also give reasonable numbers but we have no time to develop this approach.

We conclude that the potential model description of $b\bar{b}$ and $c\bar{c}$ works very well, that the potential is very well fixed in the interval 0.1 to 1 Fermi, where the two potential models coincide (after a shift due to quark mass differences), and that the potential which has the correct QCD behaviour at short distances is favoured. Also flavour independence is very well tested.

Now we want to move towards lighter quark masses, because, after all, the non-relativistic description of $c\bar{c}$ works very well as far as energies are concerned in spite of the fact that $\langle v^2/c^2 \rangle \simeq 0.25$ (E1 or M1 transition rates are not as satisfactory but relativistic corrections can improve the situation). So if it works for $c\bar{c}$ why not try to fit $s\bar{s}$ with potential (2)? Naturally, one can reproduce the ϕ mass, 1020 MeV by adjusting the s quark effective mass to be 518 MeV. However, there are checks:

1) One predicts a ϕ', radial excitation of the ϕ, 615 MeV above the ϕ, while an experimental candidate from Orsay is 630 MeV above the ϕ.

2) Once the strange quark mass is fixed, one can predict the F and the F* masses:

$$m_F = 1990 \text{ MeV} \qquad m_{F^*} = 2110 \text{ MeV}.$$

Experiments give masses for the F ranging from 2020 MeV to 1970 MeV[10].

This success has encouraged J.-M. Richard[11] to use potential (2), together with a phenomenological spin-spin force of the Fermi type adjusted to the $J\psi - \eta_c$ mass difference to calculate the mass of the Ω^- which is a sss system. The recipe giving the quark-quark potential inside a baryon which is a colour singlet is

$$V_{QQ} = \frac{1}{2} V_{Q\bar{Q}} \tag{3}$$

This is suggested by what happens for one gluon exchange or more generally colour octet exchange, and has the advantage that it is insensitive to the choice of quark masses: if one neglects changes in kinetic energies, a fit of the $Q\bar{Q}$ system with quark mass m_Q and potentia $V_{Q\bar{Q}}$ will be stable in the change

$$\begin{aligned} m_Q &\to m_Q + \Delta \\ V_{Q\bar{Q}} &\to V_{Q\bar{Q}} - 2\Delta \, . \end{aligned} \tag{4}$$

The prescription (3) is such that also the three-quark system mass will be left approximately invariant in the change (4). Anyway, J.-M. Richard has taken seriously this prescription and found

$$m_{\Omega^-} = 1665 \text{ MeV},$$

while experiment gives

$$m_{\Omega^-} = 1672 \text{ MeV}.$$

This kind of success may, of course, be fortuitous but it leads to reconsidering the systems containing even lighter quarks, i.e., up and down quarks, mesons and baryons, and trying to fit them in a non-relativistic way with an effective mass of about 300 MeV for the up and down quarks and a flavour independent potential which may or may not be constrained to satisfy property (3). In fact, in the past, this has been done to a certain extent by A. De Rújula, H. Georgi and S. Glashow[12)] and they have been incredibly successful, justifying to lowest order in $m_s - m_{u,d}$ the Gell-Mann-Okubo mass formulae for the octet and the decuplet, showing that the spin-spin force derived from one gluon exchange has the right qualitative features:

$$m_{J=1} > m_{J=0} \qquad \text{for mesons}$$

$$m_{J=3/2} > m_{J=1/2} \qquad \text{for baryons,}$$

and leads, with $m_s/m_u = 0.6$, to a lot of interesting predictions like

$$\frac{M_{K^*} - M_K}{M_\rho - M_\pi} = 0.6 \qquad (\text{exp. } 0.64)$$

$$\frac{M_{D^*} - M_D}{M_{K^*} - M_K} = 0.29 \qquad (\text{exp. } 0.35) \tag{5}$$

$$\frac{M_{\Xi^*} - M_\Xi}{M_{\Sigma^*} - M_\Sigma} = 1 \qquad (\text{exp } 1.12)$$

$$\frac{1}{2}\,\frac{2M_{\Sigma^*} + M_\Sigma - 3M_\Lambda}{M_\Delta - M_N} = 1 \quad (\text{exp } 1.04) \tag{6}$$

$$\frac{M_\Sigma - M_\Lambda}{M_\Delta - M_N} = \frac{2}{3}\left(1 - \frac{m_u}{m_s}\right) = 0.26 \quad (\text{exp } 0.26)$$

$$\frac{M_{\Sigma_c} - M_{\Lambda_c}}{M_\Delta - M_N} = 0.55 \quad (\text{exp } 0.56),$$

with $m_c = 1.8$, and naturally, the magnetic moment predictions of the quark model:

$$\mu_N/\mu_p = -0.67 \qquad (\text{exp } -0.68)$$
$$\mu_\Lambda/\mu_p = -0.2 \qquad (\text{exp } -0.22)$$
$$\mu_{\Sigma^+}/\mu_p = 0.95 \qquad (\text{exp } 0.85 \pm 0.01)$$
$$\mu_{\Sigma^-}/\mu_p = -0.38 \qquad (\text{exp } -0.50 \pm 0.09)$$
$$\mu_{\Xi^-}/\mu_p = -0.16 \qquad (\text{exp } -0.25 \pm 0.01)$$
$$\mu_{\Xi^0}/\mu_p = -0.49 \qquad (\text{exp } -0.45 \pm 0.05)$$

What is missing in the list is μ_{Ω^-} ! It could be measured and <u>this measurement would be very interesting</u> !

However, A. De Rújula et al. did something very crude: they worked to lowest order in the symmetry breaking due to the u, d and s mass differences which meant that they could use a variational approach and disregard the distortions of the space wave function due to the breaking. This might be a very bad approximation, especially if heavy charmed or beautiful quarks are present. Calculations using a flavour independent potential for mesons and baryons, and the Schrödinger equation or a modified version, in which relativistic kinematics are used, have been made by A. Bhadury et al.[13] and by D.P. Stanley D. Robson[14] first. However, it is not very easy to get a feeling from these calculations which end with long lists of numbers predicted but are sometimes criticized because of the many parameters introduced. Also, it is difficult to be certain that calculations are sufficiently accurate.

I proposed to present the results in a way in which the improvements, with respect to the "naîve" A. De Rújula et al. treatment are manifest so as to measure the effects of the wave function distorsion by symmetry breaking by the quark effective masses. For instance, the expression in the Gell-Mann-Okubo mass formula is not exactly zero and one should try to reproduce this. Similarly, one can study the deviations of the right-hand side of Eqs. (5) and (6) from unity. Two groups have done this S. Ono and F. Schöberl using variational methods with superpositions of Gaussians[15], J.-M. Richard and P. Taxil using an expansion in hyperspherical harmonics and solving a system of coupled differential equations[16] (a very efficient procedure for quark-quark potentials). Here, I report the J.-M. Richard-P. Taxil results (the others are very similar).

The Gell-Mann-Okubo mass formula for the octet gives:

$$2\,(M_N + M_\Xi) - M_\Sigma - 3M_\Lambda = -25 \pm 5 \text{ MeV}$$

experimentally, where the uncertainty is due to the spread due to electromagnetic effects.

A naïve estimate disregarding the distortion of the wave functions but only the spin-spin interaction of A. De Rújula et al. gives + 35 MeV ! However, J.-M. Richard and P. Taxil using a reasonable soft potential of the type (2) get - 22 MeV, in perfect agreement with experiment. Similarly, they get

$$\frac{M_{\Xi^*} - M_\Xi}{M_{\Sigma^*} - M_\Sigma} = 1.08 \quad (\text{exp. } 1.12)$$

$$\frac{1}{2}\,\frac{2\,M_{\Sigma^*} + M_\Sigma - 3M_\Lambda}{M_\Delta - M_N} = 1.07 \, (\text{exp } 1.04)$$

It is encouraging to see that these results all go into the right direction

Finally, I want to report another calculation of J.-M. Richard and P. Taxil to obtain the mass of the newly discovered hadron (csu) at CERN[17], with a mass of 2.46 GeV. A previous calculation by N. Isgur[18], using a harmonic oscillator potential gave 2.50 MeV for this state, a result already quite nice.

J.-M. Richard and P. Taxil[19] have taken a soft potential of the type (2) allowing for a change of the coefficients and fitting the nucleon, the Δ, the Ω^- and the Λ_c, and choosing $m_{u,d}$ = 300 MeV. They get

$$m_s = 600 \text{ MeV} \;, \quad m_c = 1895 \text{ MeV}$$

$$V_{qq} = \frac{1}{2}\left[-8.3377 + 6.9923\, r^{0.1}\right]. \tag{7}$$

Notice the constants in (7) are not very different from those in (2). Their results are listed in Table 3.

	theory	exp.		theory	exp.
N	input	0.939 GeV	Ω^-	input	1.672 GeV
Δ	input	1.232 GeV	Λ_c	input	2.283 GeV
Λ_0	1.111 GeV	1.115 GeV	Σ_c	2.443 GeV	2.450 GeV
Σ	1.176 GeV	1.193 GeV	Σ_c^*	2.542 GeV	
Ξ	1.304 GeV	1.318 GeV	A(csu)	2.457 GeV	2.460 GeV
Σ^*	1.392 GeV	1.383 GeV	S(csu)	2.558 GeV	
Ξ^*	1.538 GeV	1.533 GeV	S*(csu)	2.663 GeV	

The states A and S have spin-1/2 but correspond to different spin structures. In the limit $m_s = m_u$, A reduces to Λ_c while S reduces to Σ_c. S* has spin-3/2.

What do we conclude from these results ? A soft potential is better than a harmonic oscillator potential. However, J.-M. Richard and P. Taxil have also tried another exercise. They take three-body forces inspired by the string picture instead of two-body forces, with a potential which is the sum of the distances of the three quarks to a point minimized over the location of the point, plus a constant. They get results almost as good.

We see therefore that this kind of approach has a great predictivity but, on the other hand, it does not allow us to solve all problems concerning the nature of the interaction. We think that it should be pursued in spite of the fact that relativistic effects are disregarded. Of particular interest are, for instance, the corrections to the naîve quark model predictions for magnetic moments because at present we begin to have very precise measurements of the magnetic moments, for instance, in the case of Σ^+ [20].

REFERENCES

1) H. Krasemann and S. Ono, Nucl. Phys. B154, 283 (1979).

2) J.L. Richardson, Phys. Lett. 82B, 272 (1979).

3) W. Buchmüller, G. Greenberg and S.H.H. Tye
W. Buchmüller and S.H.H. Tye, Phys. Rev. D24, 132 (1981).

4) G. Bhanot and S. Rudaz, Phys. Lett. 78B, 119 (1978).

5) A. Martin, Phys. Lett. 100B, 511 (1981).

6) The latest data on the Υ system can be found in the review by P.M. Tuts, Electron Photon Conference, Cornell (1983), to be published.

7) D. Beavis et al., Phys. Rev. D20, 743 and 2345 (1979).

8) W. Buchmüller, Phys. Lett. 112B, 479 (1982).

9) E. Eichten and F.L. Feinberg, Phys. Rev. Lett. 43, 1205 (1979); Phys. Rev. D23, 2724 (1981).

10) S.L. Wu, Nato Advanced Study Institute on Particles and Fields, Cargèse 1983, to be published.

11) J.-M. Richard, Phys. Lett. 100B, 515 (1981).

12) A. De Rújula, H. Georgi and S. Glashow, Phys. Rev. D12, 147 (1975)

13) A. Bhadury et al., Phys. Rev. Lett. 44, 3180 (1980).

14) D.P. Stanley and D. Robson, Phys. Rev. D21, 3180 (1980);
D.P. Stanley and D. Robson, Phys. Rev. Lett. 45, 235 (1980).

15) S. Ono and F. Schöberl, Phys. Lett. 118B, 419 (1982).

16) J.-M. Richard and P. Taxil, Marseille CPT preprint (1983).

17) S.F. Biagi et al., Phys. Lett. 122B, 455 (1983).

18) N. Isgur, New Flavours and Hadron Spectroscopy, Proc. XVIth Rencontre de Moriond, (Les Arcs, France, 1981), eds. J. Tran Tranh Van (Editions Frontières, Dreux, France, 1981).

19) J.-M. Richard and P. Taxil, Phys. Lett. 128B, 453 (1983).

20) C. Ankelbrandt et al., Fermilab preprint 83/52 Exp, to appear in in Phys. Rev. Lett.