QUARKS AND THE NN- AND NN-INTERACTION

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1. INTRODUCTION

One of the main problems in the nucleon-nucleon interaction today is: "What is the influence of the quark substructure of the nucleons?" Everybody is convinced that there should be quark effects, but how do we get a handle on them? One way to try to study this problem is to look at multiquark states. There are many predictions [1 - 11] of multiquark states in many different channels. But do these multiquark states really exist? What is the evidence? In our studies of the nucleon-nucleon (NN) and antinucleon-nucleon ($\bar{N}N$) interaction we have been looking at some aspects of this problem.

In the following section we will first give a quick review of some of the predictions for multiquark states. Next we indicate the construction [12, 13] of a modified effective range function, where the long range part of the interaction is treated correctly. This modified effective range function is used for the description of the $^1S_0$ phaseshift in a multienergy phaseshift analysis of the low energy pp-data below 30 MeV

The P-matrix introduced by Jaffe and Low [14] is closely connected to this modified effective range function [15]. We made a multienergy phaseshift analysis where we parametrized this P-matrix. These P-matrix parameters can be compared with predictions for the six quark states [3,4,5]. Some other examples of very simple applications of the P-matrix are presented.

We will also look at the $\bar{N}N$-interaction. Recently a coupled channel $\bar{N}N$-potential was constructed [16] based on the Nijmegen Model D baryon-baryon potential [17]. This model D potential has been very successful in the description of the NN-data, as well as the YN-data. This $\bar{N}N$-potential gives a good fit to the presently available $\bar{N}N$ scattering data. Using backward elastic $\bar{p}p$ scattering data we present some evidence for the S(1940) meson, which could be a $Q^2\bar{Q}^2$-state.

Finally we look at another aspect of quarks. In the construction of NN-potentials from meson exchange one should take into account the quark substructure of the nucleons. In the last section we report on a preliminary estimate of quark structure effects in low-energy pp-scattering.

2. MULTIQUARK STATES

Specific predictions about multiquark states are almost all based on the MIT-bagmodel [18]. Let us first look at some of these predictions of the MIT-bagmodel, when all quarks Q (and antiquarks $\bar{Q}$) are in the lowest s-state of a spherical bag.

The mesonic multiquark states of the type $Q^2\bar{Q}^2$ were first considered by R.L. Jaffe [2] with quite interesting results. He found as lowest states a nonet of scalar mesons ($J^{PC} = 0^{++}$). The lowest meson of this nonet has $I = 0$, $M = 640$ MeV, and a flavor wave function $ud\bar{u}\bar{d}$. Because this state couples strongly to the $\pi\pi$-channel it will be wide. This state can be identified with the $\varepsilon(700)$. This wide ε-meson is very essential in the Nijmegen baryon-baryon potential [17,19]. In section 5 about some other applications of the P-matrix we will present once more some evidence for this meson. The other non-strange mesons of this scalar nonet are a degenerate $I = 0$ and $I = 1$ pair of states at $M = 1.12$ GeV. The corresponding flavor wavefunctions are for $I = 0$: $(u\bar{u} + d\bar{d})s\bar{s}$, and for $I = 1$: $u\bar{d}s\bar{s}$, $(u\bar{u} - d\bar{d})s\bar{s}$, and $d\bar{u}s\bar{s}$. These states couple strongly to the $K\bar{K}$-channel and can be identified with the S*(980) and $\delta(980)$ mesons. The mass shift from the bagmass $M = 1.12$ GeV to the observed meson mass $M = 0.98$ GeV ($K\bar{K}$ threshold) can be understood. The recognition of the states ε, S*, and δ as crypto-exotic states did solve a long standing puzzle in the old quark model, where these states were thought to be $^3P_0$ $Q\bar{Q}$-states. Now we believe [11] these $^3P_0$-mesons to appear around $M = 1.28$ GeV. We will not discuss here the baryonic multiquark states [1] of the type $Q^4\bar{Q}$, but only mention that also for these states there is some evidence [20].

Very important was the calculation [3] by R.L. Jaffe of the Q^6-states in a spherical bag [4,5]. The lowest state is an SU(3) flavor-singlet with $J^P = 0^+$, $M = 2.167$ GeV and a flavor wave function $u^2d^2s^2$. This state should appear as a bound state in the $^1S_0$ $\Lambda\Lambda$-channel about 64 MeV below the $\Lambda\Lambda$-threshold. Such a state can decay only via the weak interactions. A recent discussion about this state can be found in [21].

In the $I = 1$ NN channel Q^6-states are predicted with $J^P = 0^+$, $M = 2.24$ GeV and $J^P = 2^+$, $M = 2.36$ GeV. There is experimental evidence [22] for a $^1D_2$ resonance in pp-scattering at $M \simeq 2.17$ GeV. The mass shift from the predicted bagmass to the neighborhood of the NΔ-threshold (2.17 GeV) can be understood. However, it is unclear, if this is the predicted Q^6-state. Other interpretations [23] are also offered. Evidence for the $^1S_0$ bag state comes from P-matrix analyses. In section 4 we will give again some of this evidence.

The predictions of the multiquark states, when all quarks are in the lowest s-state of the spherical bag, are probably the strongest predictions. More speculative are the predictions of the orbitally excited states [6 - 11], which are based on the model of a fast rotating string [24]. The quarks (and antiquarks) are situated in two oppositely colored clusters each at one end of this string. The color charges of these clusters have to be complementary, such that the whole string is colorless. The string itself is formed by the color electromagnetic fields, which hold the colored clusters together.

When this model is applied to the baryon resonances, considered as a Q-Q^2 string, the results are very good [10]. This gives then some confidence for the applications to more speculative situations as the Q^2-$\bar{Q}^2$ strings [7,8,11] and Q^2-Q^4-strings [6]. With the model of the Q^2-$\bar{Q}^2$ string a very rich spectrum of baryonium states is predicted. There has lately been a very negative mood [25] about the experimental evidence for such states in $\bar{p}p$ scattering. In section 6 we will present some evidence for the S(1940) state. Even if this state exists, the exact nature of the state is of course unclear.

3. MODIFIED EFFECTIVE RANGE EXPANSION

An extensive literature [12,13,26] exists on effective range expansions in two body scattering. We will outline the method [12,27], but for clarity we will restrict ourselves to simple potentials. We assume that the potential between the two particles can be written as

$$V = V_S + V_L \quad ,$$

where the long range part V_L of the interaction is well-known. To avoid inessential complications we assume that V_L is finite at the origin ($r = 0$), and when $r \to \infty$

$$r\, V_L \sim e^{-\mu_L r} \quad .$$

We need to study the solutions of the radial Schrödinger equation for V_L

$$\left(\frac{d^2}{dr^2} - \frac{\ell(\ell+1)}{r^2} + k^2 - 2m\, V_L\right) \chi(r) = 0 \quad ,$$

where k is the relative c.m. momentum and m the reduced mass. This equation has two independent solutions.

The regular solution $\chi_R(r)$ is defined by the boundary condition at the origin

$$\chi_R(r) = \frac{r^{\ell+1}}{(2\ell+1)!!} \sum_{n=0}^{\infty} a_n\, r^n \qquad \text{with } a_0 = 1 \quad .$$

The asymptotic behavior of χ_R for $r \to \infty$ is

$$\chi_R(r) \sim \frac{N_L}{k^{\ell+1}} \sin\left(kr - \ell\frac{\pi}{2} + \delta_L\right) \quad .$$

This way we can determine for the long range potential V_L the phase shift $\delta_L(k)$ and the normalization constant $N_L(k)$. We omit in our notation the dependence of δ_L and N_L on the orbital angular momentum ℓ. The other solution, called irregular solution $\chi_I(r)$ will be defined by the boundary condition at the origin

$$\chi_I(r) = -\frac{(2\ell-1)!!}{r^\ell} \sum_{n=0}^{\infty} b_n r^n \quad ,$$

with $b_0 = 1$ and $b_{2\ell+1} = 0$. Strictly speaking this solution $\chi_I(r)$ is only irregular at the origin for $\ell \neq 0$. The asymptotic behavior of this irregular solution for $r \to \infty$ can be shown to be

$$\chi_I(r) \sim -\frac{k^\ell}{N_L} \cos\left(kr - \ell\frac{\pi}{2} + \delta_L\right) + H_L\,\chi_R(r) \quad .$$

This defines the function $H_L(k)$. We can now define the effective range function

$$X(k) = \frac{1}{N_L^2} k^{2\ell+1} \cot(\delta - \delta_L) + H_L \quad ,$$

where $\delta(k)$ is the phaseshift belonging to V. This effective range function X(k) has nice analyticity properties in the complex k^2-plane or in the E-plane. The function X(E) is regular in a large region containing the origin. It is a real analytic function, so

$$X(E) = X^*(E^*) \quad .$$

The closest righthand singularity appears for $E = E_R$ where the first inelastic threshold opens up. The closest lefthand singularity E_L is determined by the range of the potential V_S. When we assume that $r\,V_S \sim e^{-\mu_S r}$ when $r \to \infty$, then the first lefthand singularity appears for $k^2 = -(\mu_S/2)^2$ or $E_L = -(\mu_S^2/8m)$.

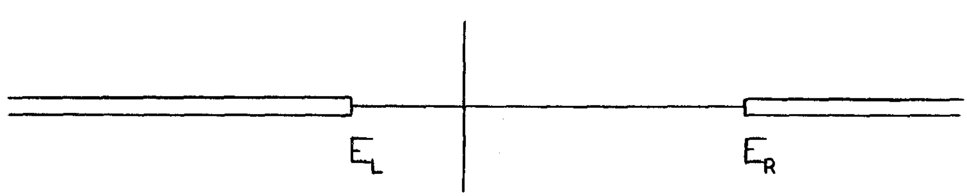


Fig. 1. Region of analyticity of X(E) in the complex E-plane.

The simplest and most well-known example of an effective range function is obtained when we take $V_L = 0$. Then we get $\delta_L = H_L = 0$, $N_L = 1$ and

$$X(k) = k^{2\ell+1} \cot \delta \quad .$$

Also well-known are the cases $V_L = V_C$ and $V_L = V_C + V_{VP}$, where V_C is the Coulomb potential and V_{VP} the vacuum polarization potential.

We have applied this method to the low energy pp-scattering in the $^1S_0$ state. We took

$$V_L = \tilde{V}_C + V_{VP} + V_\pi \quad .$$

Here $\tilde{V}_C$ is the improved Coulomb potential [28], and V_π the static one-pion-exchange potential. We used the relativistic connection between k^2 and E. When we make for $X(k^2)$ the expansion

$$X(k^2) = -\frac{1}{a_\pi} + \frac{1}{2} r_\pi k^2 + \frac{pk^4}{1 - qk^2} \quad ,$$

then we get a very good fit to the $^1S_0$ phaseshift from $T_L = 0$ to 300 MeV. This method allows a determination of the π^0-proton coupling constant.

We made [29] a multienergy phaseshift analysis of 556 low energy pp-scattering data below $T_{lab} = 30$ MeV. We used 11 parameters to describe the various partial waves. The parameter q in the modified effective range expansion was fixed by [30] $\delta(^1S_0) = 0$ at 253 MeV. We searched also for the π^0-proton coupling constant. We found $g_0^2/4\pi = 14.1 \pm 1.0$ and $\chi^2_{min} = 504.1$. From the this way determined values of a_π, r_π, p, q, and g_0^2 we calculated the more customary [31] used scattering length a_E and effective range r_E. We found $a_E = -7.804 \pm 0.002$ fm and $r_E = 2.783 \pm 0.007$ fm in agreement with earlier determined values [31].

4. THE P-MATRIX

In the P-matrix approach [14] one divides the interaction region into two parts. In the region $r > b$ one assumes that the interaction is well-known and can be described by a potential V. The interaction in the region $r < b$ will be described by a boundary condition on the radial wave function $\chi(r)$ at $r = b$. This boundary condition is called the P-matrix, where

$$P(k;b) = \left[\frac{r}{\chi} \frac{d\chi}{dr} \right]_{r=b} \quad .$$

This is essentially the boundary condition model of Lomon and Feshbach [32].

This P-matrix approach can be related to the treatment [15] of the effective range function X(k) when we choose

$$V_L = 0 \quad \text{for } r < b \quad \text{and}$$
$$V_L = V \quad \text{for } r > b \quad .$$

In this special case one can write down directly the regular and irregular solutions of V_L in the region $r < b$. They are

$$\chi_R(r) = \frac{1}{k^{\ell+1}} J(kr) \quad \text{and} \quad \chi_I(r) = k^{\ell} N(kr) \quad ,$$

where $J(x) = x\, j_\ell(x)$, $N(x) = x\, n_\ell(x)$ with $j_\ell(x)$ and $n_\ell(x)$ the spherical Bessel and Neumann functions, and $J'(x) = dJ/dx$, $N'(x) = dN/dx$.

The modified effective range function X(k) and the P-matrix are related in this case by

$$X = k^{2\ell+1} \frac{PN(kb) - kbN'(kb)}{PJ(kb) - kbJ'(kb)} \quad .$$

This shows that the functions X and P have the same domain of analyticity We have made use of this property in the following way. Instead of parametrizing the experimental data by the 4 parameters a_π, r_π, p, and q, one could just as well try to parametrize the P-matrix [33]. We used the one-pole-approximation

$$P(s;b) = c + \frac{r}{s - s_0} \quad .$$

This implies again 4 parameters: b, s_0, r, and c.

This one-pole-approximation was used [34] for the parametrization of the energy dependence of the $^1S_0$ phaseshift in an energy dependent phaseshift analysis of the 556 pp-scattering data below T_{lab} = 30 MeV. For the potential V_L in the region $r > b$ we used the sum of the improved Coulomb potential $\tilde{V}_C$, the vacuum polarization potential V_{VP} and the static one-pion-exchange potential V_π. The higher partial waves were parametrized with 8 parameters. Because the low energy data do not determine b very well, we used some of Arndt's s-wave phaseshifts at higher energies to determine b. The other parameters were determined by

Table 1: P-matrix parameters.

	b(fm)	$\sqrt{s_0}$(GeV)	r (GeV^2)	c
P. Mulders [35]	1.18	2.34	4.8	
	1.28	2.25	5.4	
	1.38	2.17	4.9	
Y. Simonov [33]	1.44	2.11	2.7	2.95
our fit	1.61	2.08	3.36	4.51
bagmodel [14,34]	1.48	2.24	0.84	

the low energy data and gave a good fit to these data (χ^2_{min} = 504.1). This fit gives $g_0^2/4\pi$ = 14.1 ± 1.0.

In table 1 we compare our results with those of other analyses [33, 35] and with the predictions of the MIT-bagmodel. The agreement with the bagmodel predictions for the radius b and for the pole position is reasonable. Our result prefers a somewhat larger bagradius. The residue at the pole is, however, in disagreement. For a discussion, see [36].

5. SOME OTHER APPLICATIONS OF THE P-MATRIX

The relation between the P-matrix and the S-matrix is very simple when $V_L \equiv 0$. Then

$$S = - \frac{P\,H_2(kb) - kb\,H_2'(kb)}{P\,H_1(kb) - kb\,H_1'(kb)} \quad ,$$

where $H_1(x) = x\,h_\ell^{(1)}(x)$ and $H_1' = dH_1/dx$. The S-matrix has a pole when

$$P = \left[\frac{x\,H_1'(x)}{H_1(x)} \right]_{x = kb} \quad .$$

For s-waves this becomes P = ikb. When we have parametrized a set of data with the help of the P-matrix, we can then determine very easily the position E_R of the S-matrix pole. It is interesting to apply this method to some well-known cases.

(i) The phaseshifts in the $J^P = \frac{3}{2}^+$ partial wave in $\pi^+ p$ scattering around the Δ^{++} are well-known [37]. The best fit with $V_L = 0$ was obtained with the parameters b = .88 fm and $\sqrt{s_0}$ = 1.38 GeV. The pole in this P-matrix lies much higher than the standard Δ^{++}-mass M = 1.232 GeV. For the position of the S-matrix pole we find

$$E_R(\Delta^{++}) = (1.21 - i\,0.05)\ \text{GeV} \quad .$$

This should be compared with the value [38]

$$E_R(\Delta^{++}) = (1210.6 \pm 0.5) - i(49.7 \pm 0.3)\ \text{MeV} \quad .$$

We find the agreement very good for such a simple approximation.

(ii) The phaseshifts in the I = J = 1 partial wave in $\pi^+\pi^-$-elastic scattering around the ρ are also well-known [39]. Our best fit (χ^2/datapoint ≈ 3) to these phaseshifts was obtained for b = 0.91 fm and $\sqrt{s_0}$ = 850 MeV. Again we notice that the P-matrix pole lies much higher than the standard ρ-mass M = 776 MeV. The S-matrix pole we found at

$$E_R(\rho) = (767 - i\,65)\ \text{MeV} \quad .$$

(iii) Also the $\pi\pi$-phases in the I = J = 0 partial wave are known [39]. Discrepancies between phaseshift analyses exist. We used the Estabrooks et al. s-channel data. A best fit to these data was obtained for b = 0.70 fm and $\sqrt{s_0}$ = 850 MeV. This is the ε-meson. The S-matrix pole we find at $E_R(\varepsilon)$ = (760 - i 200) MeV. The data of Biswas et al. [40] give the same result, only the width comes out smaller. The Estabrooks et al. t-channel data give very different results.

6. AN $\bar{N}N$-POTENTIAL AND THE S(1940) MESON

Recently we constructed a coupled channel $\bar{N}N$-potential [16] starting from the Nijmegen Model D baryon-baryon potential [17]. This Nijmegen D potential gives good fits to the NN-scattering data, the YN scattering data, and the binding energies of the Λ and Σ in infinite nuclear matter [41]. This meson-theoretical potential is cut off in the inner region and there a phenomenological potential has been added. The annihilation is described by purely phenomenological potentials. The total number of free parameters used in the construction of this potential is 14. We fit 970 scattering data up to T_{lab} = 482 MeV with χ^2/datapoint = 1.49. Especially the fit to the elastic $\bar{p}p$-scattering data is good at about all angles and energies. Very interesting is the fit to the $\bar{p}p$ backward elastic cross section [42] between p_{lab} = 406 MeV/c and p_{lab} = 922 MeV/c. This fit is shown in Fig. 2. The total χ^2 of the fit to these 30 data-points is χ^2 = 27.0. Although this appears to be quite an acceptable χ^2, one observes that the 6 neighboring points from p_{lab} = 486 MeV/c to p_{lab} = 534 MeV/c all lie above the theoretical curve and they contribute

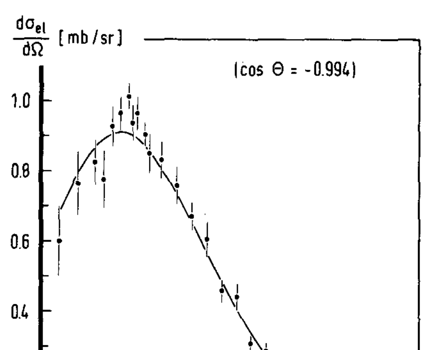


Fig. 2: The $\bar{p}p$ backward elastic cross section. Experimental points from [42].

together 13.0 to the total χ^2 of 27.0. We tried to test the statistical significance of this deviation for these 6 neighboring points as follows First we would like to be reasonably sure that the experimental points (except for those 6) are spread around the theoretical curve according to a Gaussian probability function. The Pearson-χ^2 test [43] indicates that this is indeed the case with about 99% reliability. Assuming then such a Gaussian probability destribution for all 30 points, the probability to find a set of 6 points on a row with these (or larger) deviations from the curve is less than 0.5%. This suggests that there is indeed a statistically significant structure in these backward elastic data around p_{lab} = 509 MeV/c ($\sqrt{s}$ = 1940 MeV). When we blame this effect on the existence of a resonance, we can estimate the parameters of this resonance. We find

$$E_R = 1935 \pm 3 \text{ MeV} \qquad \text{and} \qquad \Gamma = 7.0 \pm 3 \text{ MeV} \quad .$$

These values for E_R and Γ are found by using a parametrization [44] of the elastic part of the S-matrix for a multichannel resonance in the presence of a background.

In a certain partial wave the elastic part of S is written as

$$S_{el} = S_{bg} + z\,(S_R - 1) \quad .$$

Here $S_{bg} = \eta\, e^{2i\delta}$ describes the elastic background, $S_R = [E - E_R - i\,\Gamma/2]/[E - E_R + i\,\Gamma/2]$ is the Breit-Wigner form, and $z = \frac{1}{2}\, e^{2i\delta}\,[\sqrt{(1+\eta)\Gamma_{el}/\Gamma} + i\,\sqrt{(1-\eta)\Gamma'_{in}/\Gamma}]^2$.

We write the total width Γ as

$$\Gamma = \Gamma_{el} + \Gamma'_{in} + \Gamma''_{in} \quad ,$$

where Γ_{el} is the partial width to the elastic channel, Γ'_{in} is the partial width to the (effective) channel to which also the elastic channel is coupled, and Γ''_{in} is the partial width to those (effective) channels to which the resonance is coupled, but from which the elastic channel is (effectively) decoupled.

In our model we claim that we can calculate the background (η and δ). To describe a resonance in a certain partial wave we have still four parameters (E_R, Γ, Γ_{el}, Γ'_{in}) to be adjusted. It then turns out that almost every partial wave can accommodate a resonance, such that the fit to the backward elastic cross section is much improved. We get typically $\chi^2 \lesssim 16$ for 30 datapoints. The values of E_R and Γ are almost independent of the partial wave in which we assume the resonance, whereas the values of Γ_{el} and Γ'_{in} vary widely.

We also tried to fit the structure in the backward cross-section using a coupled channels model. This consisted of the before mentioned

$\bar{N}N$-coupled channels model, supplemented with one additional confined channel. The idea behind this approach is, that $\bar{N}N$-scattering might show the existence of Q^2-$\bar{Q}^2$ states [7,8,11]. From the model of the fast rotating string for such states we take over the idea of a linear confining potential of the form

$$V = A + Br$$

with B = 972 MeV/fm, as well as an estimate M = 730 MeV/c^2 for the mass of the diquark system. Moreover we require an orbital angular momentum L unequal to zero for these Q^2-$\bar{Q}^2$ states.

The reaction $\bar{N}N \to \bar{Q}^2$-Q^2 can be described by the quark pair creation or $^3P_0$-model. This $^3P_0$-model suggests a transition potential between the $\bar{N}N$-channel and the $\bar{Q}^2$-Q^2 channel of the form

$$V_S(r) \sim r\ e^{-(mr)^2} \quad .$$

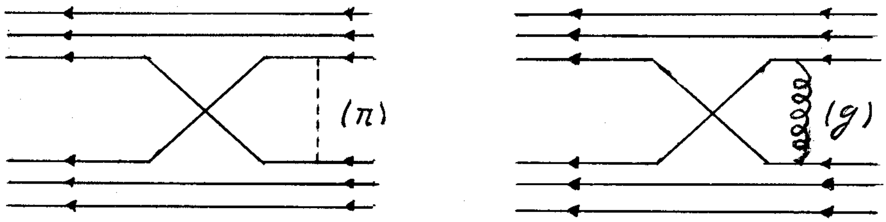
This form for the transition potential we have taken over in our simple model. Also in this case the data are not sufficient to constrain the resonance to a particular partial wave. For example we coupled the $\bar{N}N$ $^{11}P_1$-wave to a confined channel with relative orbital angular momentum L = 2. The constant term A in the potential is chosen such that the L = 2 bound state appears at 1940 MeV.

In this example the χ^2 for the 30 points is lowered from 27.0 to 15.9, σ_T decreases somewhat (maximally about 2.1 mb), and σ_{CE} increases (at maximum 0.32 mb). These changes in the cross section are about at the level of the current experimental error. In the future we can expect much more accurate results from LEAR, which might then well resolve the question whether the S-meson exists or not.

7. QUARK-ANTISYMMETRIZATION CORRECTIONS TO THE OBE-POTENTIALS

In low energy pp-scattering there are some discrepancies between recent phase shift analyses and the predictions of the better potential models [45]. For example the Nijmegen soft core OBE-model [19] fits the central combination Δ_C of the ^3P-phase shifts excellently, but for the tensor combination Δ_T a discrepancy exists between the model and experiment [46]. It appeared that this problem could not be solved by a simple refit of the model due to constraints imposed by the $^1S_0$- and the Δ_C-phase shifts.

In looking for an explanation of the discrepancies in $\Delta_T(^3P)$ in terms of quark structure effects we found that we can account for the differences by calculating the effects of the following processes



(a) (b)

Process (a) is a correction to the OPE-potential which improves $\Delta_T(^3P)$. Process (b) is the contribution of gluon exchange, which in our calculation serves to cancel the central potential contribution from (a) to preserve $\delta(^1S)$ and $\Delta_C(^3P)$.

In this section we report briefly on our estimate of quark structure effects in low energy pp scattering. A more detailed version will be published elsewhere [47].

In the evaluation of the effect of (a) and (b) we use the resonating-group-method [48] formalism in a form as it has been applied to nucleon-nucleon scattering by Ribeiro [49] and Oka and Yazaki [50]. These authors calculated the contribution from (b). In diagram (a) and (b) the solid lines denote constituent quarks and their distribution in each nucleon is supposed to be provided by the nonrelativistic harmonic oscillator ground state wave function. The gluon and also the bosons π, ρ, ε, ... are supposed to couple directly to the quarks in the nucleons. We describe the nucleon at position $\underline{r}$ by

$$\phi(\underline{r}) = \int \left(\prod_{i=1}^{3} d^3 r_i \right) \delta^3\left(\underline{r} - \frac{1}{3} \sum_{i=1}^{3} \underline{r}_i\right) \varphi(\underline{r}_1,\underline{r}_2,\underline{r}_3)\ \varphi_{FS}\ \varphi_C \quad ,$$

where φ_{FS} and φ_C are the flavor-spin (SU(4)) and color (SU(3)) wave-functions respectively. The quark distribution is described by

$$\varphi(\underline{r}_1,\underline{r}_2,\underline{r}_3) = (\pi\ \sqrt{3}\ R^2)^{-3/2} \exp\left[- \frac{1}{6R^2} \sum_{i<j} (\underline{r}_i - \underline{r}_j)^2\right] \quad ,$$

where R is the radius of the nucleon.
In the resonating group method the two-nucleon wave-function is

$$\psi(\underline{r}_1,\underline{r}_2) = \quad \{\chi(\underline{r}_1 - \underline{r}_2)\ \phi_1(\underline{r}_1)\ \phi_2(\underline{r}_2)\}$$

where   is the antisymmetrizer (see [49]) and $\chi(\underline{r})$, with $\underline{r} = \underline{r}_1 - \underline{r}_2$, is the relative wave function. Variation with respect to the relative wave function leads to the integro-differential equation [51]

$$(-\Delta/2\mu + V(\underline{r}))\,\chi(\underline{r}) + \int K(\underline{r},\underline{r}')\,\chi(\underline{r}')\,d^3r' = E\,\chi(r)$$

where μ = M/2, M = nucleon mass, and E = kinetic energy in cm-frame. In this equation $V(\underline{r})$ stands for the local potential (OBE-potential) and the non-local kernel $K(\underline{r},\underline{r}')$ contains all terms from the overlap

$$K(\underline{r}',\underline{r}) = \langle\psi'(\underline{r}_1',\underline{r}_2')|A(H-E)|\psi(\underline{r}_1,\underline{r}_2)\rangle$$

where A = - 1 . The energy overlap $\langle\psi'|A(H_0-E)|\psi\rangle$ is small in particular at low energies and we neglect it henceforth. So

$$K(\underline{r}',\underline{r}) \simeq \langle\psi'|AV|\psi\rangle$$

which defines the effects due to the quark structure of the nucleons in the present treatment. In fact one can view $K(\underline{r}',\underline{r})$ as a correction to the OBE-potential plus the effect of one-gluon exchange. In principle $K(\underline{r}',\underline{r})$ contains besides (a) also contributions from ρ, ω, ε, etc. It appeared that at low energies these latter contributions are much smaller than those from π-exchange and we neglect them. Also one gets contributions where the gluon and the mesons couple to quarks that are not operated on by A, but these are much smaller than (a) and (b) and so we neglect these too. In the calculation of $K(\underline{r}',\underline{r})$ for (a) and (b) we use the following quark-quark interactions.

$$V_{ji}(\pi) = (f^2/4\pi)(\underline{\tau}_j\cdot\underline{\tau}_i)\left[\frac{1}{3}(\underline{\sigma}_j\cdot\underline{\sigma}_i)\,\phi(\underline{r}_{ij}) + S_{ij}\,\chi(r_{ij})\right]$$

$$V_{ji}(g) = (\lambda_j\cdot\lambda_i)\left[a\,\underline{r}_{ij}^2 + b\,(\underline{\sigma}_j\cdot\underline{\sigma}_i)\right] \quad .$$

In $V_{ji}(\pi)$ $\phi(r)$ and $\chi(r)$ are the usual spin-spin and tensor pion-exchange potential functions $\phi(r) = \exp(-mr)/mr$ and $\chi(r) = \frac{1}{3}\left(1+\frac{3}{mr}+\frac{3}{(mr)^2}\right)\phi(r)$ In $V_{ji}(g)$ the central and spin-spin term are taken similar as those of Ribeiro [49]. For the evaluation of $K(\underline{r}',\underline{r})$ we make the following approximation in the pion propagator

$$(\underline{K}^2 + m_\pi^2)^{-1} \simeq \sum_{\alpha=1}^{2} a_\alpha \exp(-\underline{K}^2/m_\alpha^2)$$

in order to be able to carry out analytically the internal integration, which occurs implicitly in $K(\underline{r}',\underline{r})$. Here the parameters are $a_1 = 3/4$, $m_1 = m_\pi$ and $a_2 = 1/4$, $m_2 = 3\,m_\pi$. Then we find for the nonlocal kernel

(i) π-exchange (a):

$$K_{ji}^{(\pi)}(\underline{r}',\underline{r}) = \sum_{\alpha=1}^{2} a_\alpha \frac{3^{9/2}}{2^8\,\pi^2}\left(\frac{f^2}{4\pi}\,\frac{m_\alpha^3}{m_\pi^2\,R^3}\right)(\tau_j\cdot\tau_i)P_{ji}\Big[(1+3\,m_\alpha^2\,r_{12}^2)$$

$$\times\,(\underline{\sigma}_j\cdot\underline{\sigma}_i) + 9\,m_\alpha^2\left((\underline{\sigma}_j\cdot\underline{r}_{12})(\underline{\sigma}_i\cdot\underline{r}_{12}) - \frac{1}{3}\underline{r}_{12}^2\,(\underline{\sigma}_j\cdot\underline{\sigma}_i)\right)\Big]$$

$$\times\,\exp\left[-(15+9\,m_\alpha^2\,R^2)\,\frac{r'^2+r^2}{16\,R^2}\right]\,\exp\left[\frac{9}{8}\,(1+m_\alpha^2\,R^2)\,\frac{\underline{r}'\cdot\underline{r}}{R^2}\right]$$

where $\underline{r}_{12} = \underline{r}' - \underline{r}$ and P_{ji} is the quark interchange operator.

(ii) g-exchange (b):

$$K_{ji}^{(g)}(\underline{r}',\underline{r}) = - \left(\frac{3}{4\,\pi^{1/3}} \right)^{9/2} \frac{1}{R^3} (\lambda_j \cdot \lambda_i)\, P_{ji} \left[- \frac{9}{4}\, a\, r_{12}^2 + b\, (\underline{\sigma}_j \cdot \underline{\sigma}_i) \right]$$

$$\times \exp \left[- \frac{15}{16\, R^2} (r'^2 + r^2) \right] \exp \left[\frac{9}{8\, R^2} (\underline{r}' \cdot \underline{r}) \right] \quad .$$

The total kernel is given by

$$K_{ji}(\underline{r}',\underline{r}) = K_{ji}^{(\pi)}(\underline{r}',\underline{r}) + K_{ji}^{(g)}(\underline{r}',\underline{r}) \quad .$$

The final steps in the evaluation of the corrections to the phase shifts are (i) the performance of the radial integrals using Coulomb wave functions, (ii) the evaluation of the $SU(4)_{FS}$ and $SU(3)_C$ matrix elements of the operators $(\lambda_j \cdot \lambda_i)P_{ji}$ etc. occurring in $K_{ji}^{(g)}$ and $K_{ji}^{(\pi)}$, (iii) partial wave expansion. We skip here the details of (i) - (iii) and will only give the final results.

As to (ii) we note that in the "intermediate configurations", which occur between the gluon or pion exchange and the quark pair exchange in (a) and (b), we must include all isospin, spin, and color configurations. We did not solve the integro-differential equation in our estimate discussed here, but evaluate the effect of the kernel $K(\underline{r}',\underline{r})$ in the Distorted-Wave-Born-Approximation (DWBA). This is reasonable at low energies. The correction to the phase shift is

$$\delta\, \Delta_\ell = - \frac{M}{p} \int_0^\infty dr_1 \left[r_1\, F(pr_1) \int_0^\infty dr_2\, K(r_1,r_2)\, F(pr_2) \right]$$

where p denotes the cm-momentum and $F_\ell(pr)$ are the Coulomb wave functions.

We find the following results for the tensor phase shifts $\Delta_T(^3P)$:

T_L(MeV)	6.14	10.00	16.00
$\Delta_T(\mathrm{OBE})^{(a)}$	- 0.537	- 0.989	- 1.640
$\delta\,\Delta_T$ (b)	+ 0.050	+ 0.104	+ 0.202
Δ_T(tot)	- 0.487	- 0.885	- 1.438
$\Delta_T(\exp)^{(c)}$	- 0.506	- 0.837	- 1.549
	± 0.160	± 0.054	± 0.051

(a) Nijmegen potential [19]

(b) a = 100 MeV fm^{-2}, b = 67.5 MeV

(c) phase shift analysis [46].

We observe a movement into the right direction.

Finally we note that we have treated b as an adjustable parameter in order to bring about the approximate cancellation of (a) and (b) in the central phase shifts. We found a value which is considerable smaller than that in [49]. It could be argued that this is a signal of the relevance of other quark cluster wave functions than those contained in $\psi(\underline{r}_1,\underline{r}_2)$ (see above) [52] for the description of scattering processes.

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