BOSON-MEDIATED INTERACTIONS BETWEEN STATIC SOURCES

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As we have seen in previous presentations to this symposium, today's framework for nuclear theory includes a short-range interaction dominated by quarks and gluons, which obey the laws of quantum chromodynamics, and a long-range region in which the interaction is carried by pions. The long-range interaction may or may not be carried also by heavier mesons; this point is controversial, and it will be some time before the role of the heavier mesons is settled. Dr. Machleidt's talk[1] showed that the trend is toward the replacement of at least some of the heavy-meson exchange interaction by multi-pion exchange interactions of various types. At present it seems that the pionic interaction will continue to be an essential part of the interaction between nucleons. It is important to understand the pionic nucleon-nucleon interaction quantitatively so that we can determine the nature of whatever other interaction is required by the nucleon-nucleon data; presumably the nonpionic interactions must be due to quarks and gluons (and possibly heavy-meson exchange). The exact role of the delta resonance is also a question; uncertainty here leads to corresponding uncertainty in the theoretical treatment of pionic interactions. The talk by Dr. Miller[2] showed that it is most attractive to treat the delta as a distinct state of the three quarks that constitute the nucleon, although any model of the delta naturally includes a substantial nucleon-plus-pion field component.

The theoretical treatment of mesonic nuclear interaction has been of two types, one that is indirect, and one that is direct. The indirect treatment first converts the meson field theory into a nucleon-nucleon potential (and a nucleon-nucleon-nucleon three-nucleon potential, a four-nucleon potential, etc.) and then uses the potentials to compute nuclear properties like nucleon-nucleon scattering. This approach is exemplified by the work of the Bonn,[1] Nijmegen,[3] and Paris[4] groups. It has the advantage that the parameters of the theory are severely restricted by being required to reproduce the extremely accurate two-nucleon data in a reasonable way.

In multinucleon systems a more direct approach has been used, in which the meson quantum field is treated as an essential component of the state vector of the system. The underlying Hamiltonian is taken to be of the form

$$
\begin{aligned}
H &= T_F + T_B + H_Y + H_Y^+ \\
T_F &= \int t(\vec{p})\, \psi^+(\vec{p})\psi(\vec{p})\, d\vec{p} \\
T_B &= \sum_\lambda \int \omega(k)\, a_\lambda^+(\vec{k})a_\lambda(\vec{k})\, d\vec{k} \\
H_Y &= -g \sum_\lambda \int a_\lambda^+(\vec{k})\, \psi^+(\vec{p})W_\lambda(\vec{k},\tfrac{\vec{p}+\vec{q}}{2})\psi(\vec{q})\, \delta(\vec{p}+\vec{q}-\vec{k})\, d\vec{p}\, d\vec{q}\, d\vec{k} \quad .
\end{aligned}
\tag{1}
$$

Since the nucleon is no longer to be regarded as an elementary particle it is reasonable to use a noncovariant form for the Hamiltonian. Thus, the form factor $W_\lambda(\vec{k},\vec{K})$ can be chosen to be such that there are no infinities in the system. From the talks that we have heard on lattice gauge theories, it seems likely that in the reasonably near future the pion-nucleon form factors $W_\lambda(\vec{k},\vec{K})$ will be provided to us by the computational results of those theories. In treating Hamiltonians like that given above, the Stanford[5] and Brooklyn[6] groups have used what is basically a mean-field technique to treat scalar and vector meson fields. Since there is no pion mean field in even nuclei, a generalization of the mean-field theory is required in order to treat the pion case; this is provided by the recently developed[7] "meson-nucleon shell model." The direct techniques have the advantage of being able to deal with strong coupling of the meson fields, but they have not yet been applied to the two-nucleon system, so that the parameters of the direct approach have not been constrained by the two-nucleon data.

The aim of the present work is to apply the techniques of the direct approach to meson-nucleon states to the treatment of the potential between static sources of meson field, with a view to testing the validity of some of the approximations used in the calculatons of the indirect potentials between nonstatic sources. In the case of static sources, new methods have been developed that permit a reasonably exact treatment of the meson field for any coupling strength, and comparison with one-meson-exchange interactions can be made explicit.

The static sources have the distributions $\rho_o(\vec{r}-\vec{R}_1)$ and $\rho_o(\vec{r}-\vec{R}_2)$, where $\rho_o(r)$ is the intrinsic density distribution of a single source, taken to be spherically symmetric. Then for isovector scalar meson field interacting with the sources, the Hamiltonian of (1) becomes

$$H = T_B + H_I + H_I^{+} \quad , \tag{2}$$

where T_B is given in Eq. (1) and

$$H_I = - g \sum_{p=1}^{2} \sum_{\lambda} \tau_\lambda^p \int a_\lambda^{+}(\vec{k}) \frac{\tilde{\rho}_o(k)}{[16\pi^3\omega(k)]^{1/2}} e^{-i\vec{k}\cdot\vec{R}_p} d\vec{k} \quad , \tag{3}$$

where $\tilde{\rho}_o(k)$ is the Fourier transform of $\rho_o(x)$, and τ^p is the isospin operator for source p.

The ground-state energy $E_g(R)$ of the Hamiltonian depends on the magnitude R of the source separation $\vec{R}$,

$$\vec{R} = \vec{R}_1 - \vec{R}_2 \quad . \tag{4}$$

Once $E_g(R)$ is determined, the static meson potential is given by

$$V(R) = E_g(R) - 2E_1 \quad , \tag{5}$$

where E_1 is the ground-state energy of a single source interacting with the meson field. As will be seen, the magnitude of V(R) is often of the order of up to a few percent of the magnitude of $E_g(R)$ over much of the range of R values of interest, so it is important to have accurate methods for calculating E_g and E_1. The case of isovector-scalar meson field may be regarded as a testing ground for developing accurate techniques that can be applied later to the more difficult case of pion mediating field.

The static meson potential computations involve five basic ideas: they are (1) variational calculations based on (2) decomposition of the meson field operator into mutually commuting internal and external parts; (3) a restricted set of one or two internal modes is chosen in a particular way so as to give (4) correct asymptotic behavior of the static meson potential; in all cases (5) accurate strong-coupling techniques are used to generate the trial variational states.

The decompositon of the meson field operator $a(\vec{k})$ into internal and external parts

$$a(\vec{k}) = a_{int}(\vec{k}) + a_{ext}(\vec{k}) \tag{6}$$

with

$$\begin{aligned} a_{int}(\vec{k}) &= \sum_{i}^{N} A_i \, \phi_i(\vec{k}) \\ (\phi_i, a_{ext}) &= 0 \, , \quad i = 1,2,\ldots N \\ (\phi_i, \phi_j) &= \delta_{ij} \\ [A_i, A_j^+] &= \delta_{ij} \end{aligned} \tag{7}$$

has been discussed at some length in references 7-10. The L_2 functions $\phi_i(\vec{k})$ are the internal mode functions or wave functions of the meson field; some specific forms are given below. The variational calculations are carried out within a subspace of the full Hilbert space; the subspace is generated by the internal-mode creation actors A_i^+ acting on the bare source state. The effective Hamiltonian in this subspace is H_{int}, where H_{int} is just H with each $a(\vec{k})$ or $a^+(\vec{k})$ replaced by $a_{int}(\vec{k})$ or $a_{int}^+(\vec{k})$. For the case of a single source, it was shown in reference 11 that a single meson mode really does give an adequately accurate approximation to the single-source ground state. Procedures for computing corrections due to the external meson field have been given in references 8 and 10.

When a single mode is used for the single source and two modes are used for the case of two sources, it has recently been shown[12] that the correct asymptotic behavior

$$V(R) \to - \frac{g_R^2}{4\pi} \tau_1 \cdot \tau_2 \int \rho_1(\vec{r}_1) \frac{e^{-m|\vec{r}_1 - \vec{r}_2|}}{|\vec{r}_1 - \vec{r}_2|} \rho_2(\vec{r}_2) \, d\vec{r}_1 \, d\vec{r}_2 \tag{8}$$

is obtained as R becomes large. This demonstration shows that the static meson potentials as computed are valid for large R and supports the use of only two modes for other values of R. The same technique that gives the correct asymptotic potential also gives an interesting variational "distorted-field" approximation (DFA) to the potential at any value of R.

In a previous computation of static meson potentials[13] the asymptotic behavior of the potential was not treated correctly; an unjustified ad hoc procedure was used to determine the zero of the potential.

The strong-coupling techniques are based on the use of coherent meson pairs[11] in the variational state vectors; this method allows an accurate energy to be computed with a fraction of the number of state components that would otherwise be required. Strong-coupling techniques are required for treating the single source and therefore also for two sources even in the asymptotic region. In the region of small R, special strong-coupling states appropriate to two isospin-$\frac{1}{2}$ sources have been used.

For the Hamiltonian of Eq. (3), the appropriate mode for the case of a single source at the origin has been shown in references 8 and 14 to be

$$\phi(k) = \frac{1}{G\omega(k)} \frac{g\tilde{\rho}_o(k)}{[16\pi^3\omega(k)]^{1/2}} \quad , \tag{9}$$

where G is both the normalization constant for the mode ϕ and a "normalized coupling constant," since with (9) it follows that

$$H_I = - G \sum_{\lambda p} \tau_\lambda^p \int \omega(k)\ a_\lambda^+(\vec{k})\ \phi(k)\ e^{-i\vec{k}\cdot\vec{R}_p}\ d\vec{p} \quad ; \tag{10}$$

comparison with T_B shows that G measures the strength of H_I relative to the strength of T_B. The static-model calculations of the 1950's used the form

$$\frac{1}{G(\omega(k) + \lambda)} \frac{g\tilde{\rho}_o(k)}{[16\pi^3\omega(k)]^{1/2}} \quad , \tag{11}$$

but it has since been shown that the appropriate value of λ in this case is zero.[8,10,15,16]

When there are two sources present, the minimum number of modes required to obtain the independent-source modes asymptotically is two, one symmetric and one antisymmetric. These are assumed to have the forms

$$\phi_\pm(\vec{k}) = \frac{1}{n_\pm\sqrt{2}}\,\phi(k)\,\left[\, e^{-i\vec{k}\cdot\vec{R}_1} \pm e^{-i\vec{k}\cdot\vec{R}_2} \,\right] \quad , \tag{12}$$

where $\phi(k)$ is the single-source mode function of (9), and the mode normalization functions $n_\pm(R)$ are given by

$$\begin{aligned} n_\pm(R) &= 1 \pm \int |\phi(k)|^2 \cos\vec{k}\cdot\vec{R}\; d\vec{k} \\ &= 1 \pm \frac{1}{R}\int \sin kR\; |\phi(k)|^2\; k dk \quad . \end{aligned} \tag{13}$$

Corresponding to the modes $\phi_\pm$ there are annihilation operators $A_\pm$. It is also useful to introduce the $\tau_\pm$ operators

$$\tau_\pm = \frac{1}{\sqrt{2}}\left(\tau^{(1)} \pm \tau^{(2)}\right) \quad . \tag{14}$$

In the asymptotic large-R region, the distorted normal modes that are orthogonal and go over into single-source modes as $R\to\infty$ are ϕ_1 and ϕ_2, where

$$\phi_\pm(\vec{k}) = \frac{1}{\sqrt{2}}\left(\phi_1(\vec{k}) \pm \phi_2(\vec{k})\right) \quad . \tag{15}$$

The mode ϕ_1 differs from $\phi(k)e^{-i\vec{k}\cdot\vec{R}_1}$ because of the factors $n_\pm$ in Eq. (12). Corresponding to ϕ_1 and ϕ_2 are the annihilation operators A_1 and A_2:

$$A_\pm = \frac{1}{\sqrt{2}}\left(A_1 \pm A_2\right) \quad . \tag{16}$$

For the source form factor $\tilde{\rho}_o(k)$ it is possible to make various choices. The particular choice

$$\tilde{\rho}_o(k) = \frac{1}{\left(1+\frac{k^2}{\Lambda^2}\right)} \tag{17}$$

has been used in the work presented here. This choice is suggested by the nonrelativistic limit of the covariant scalar source density. It is also possible to use a cloudy-bag source function or a function with a sharp cutoff. The requirement is only that no divergent integrals be obtained. As was mentioned earlier, it is expected that the relevant form factors will eventually be computed from lattice gauge theory.

The above choices of meson modes and internal field lead to corresponding internal Hamiltonians. For the single source, the internal Hamiltonian is

$$H_{int}^{(1)} = W \left[A_1^{\dagger}\cdot A_1 - G\tau^{(1)}\cdot(A_1^{\dagger} + A_1) \right] , \tag{18}$$

where W is the expectation value of the meson energy $\omega(k)$ in the meson mode $\phi(k)$,

$$W = \int \omega(k) \; |\phi(k)|^2 \; d\vec{k} \quad . \tag{19}$$

Despite the apparent simplicity of $H_{int}^{(1)}$, its eigenvalues have so far only been determined numerically. When there are two sources, the Hamiltonian is most simply written in terms of $A_{\pm}$:

$$\begin{aligned} H_{int} &= \sum_{s=\pm} W_s \left[A_s^{\dagger}\cdot A_s - Gn_s\tau_s\cdot(A_s^{\dagger}+A_s) \right] \\ W_s &= \int \omega(k) \; |\phi_s(\vec{k})|^2 \; d\vec{k} \quad , \end{aligned} \tag{20}$$

but for the asymptotic region the description in terms of A_1 and A_2 is more useful:

$$H_{int} = \sum_{p=1}^{2} H^p(R) + H_{12}(R)$$

$$H^p(R) = \frac{W_+ + W_-}{2} \left[A_p^{\dagger}\cdot A_p - G(R)\tau_p\cdot(A_p^{\dagger}+A_p) \right]$$

$$G(R) = G\,\frac{n_+W_+ + n_-W_-}{W_+ + W_-} \tag{21}$$

$$H_{12} = \frac{W_+ - W_-}{2}\left[A_1^{\dagger}\cdot A_2 + A_2^{\dagger}\cdot A_1\right] - G\,\frac{n_+W_+ - n_-W_-}{2}\left[\tau_1\cdot(A_2^{\dagger}+A_2) + \tau_2\cdot(A_1^{\dagger}+A_1)\right] \quad .$$

Since the modes $\phi_{\pm}$ depend on R, it follows that $W_{\pm}$ and H_{int} also depend on R.

For a given separation of the two sources, H_{int} can be diagonalized over a selected subspace of the Hilbert space. Figures 1 and 2 show the results of some calculations for the case that $g^2/4\pi$ is 1.0 and the cutoff Λ in (17) is taken to be 7m, where m is the meson mass. The computed value of G is .735; the renormalized value of $g^2/4\pi$ is .245. The value of E_I is -5.88m, so that $2E_I$ takes the rather large value of -11.76m. In both of these figures, the solid curve shows the result of using the form (21) for H_{int} with a single state that is the product of the ground states of $H^{(1)}(R)$ and $H^{(2)}(R)$; this is the "distorted-field" approximation to the potential (DFAP). The dashed curve in both figures shows the short-range potential (SRP), which is the result of diagonalizing H_{int} in the form given by Eq. (20) over subspaces with three states for T=0 and six states for T=1. Each state in the subspace is a coherent-pair state[11] constructed on a basic state. The basic states include all basic states with up to two mesons. Coherent pairs are used for both the + and − modes. The dot-dashed curve in both figures is the renormalized one-meson-exchange potential (ROMEP). As was shown in reference 12, the DFAP is asymptotically equal to the ROMEP.

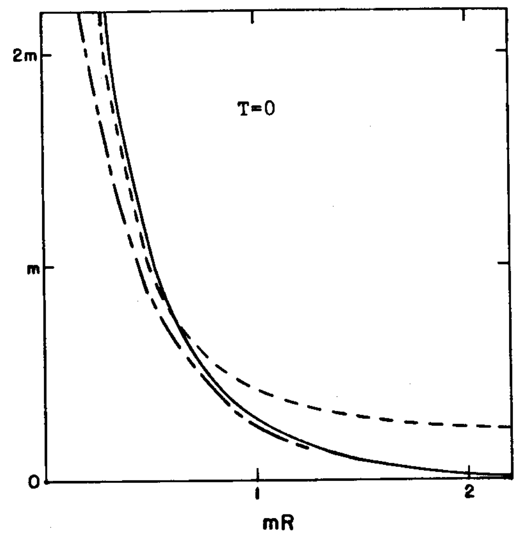

Figure 1

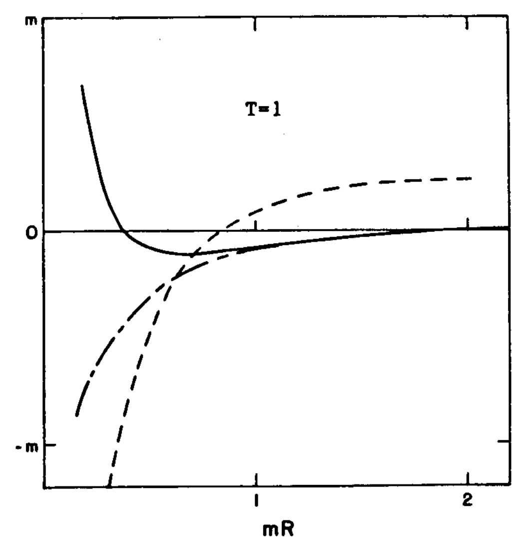

Figure 2

Note that the ROMEP is not a variational result, so that it is not a candidate for the potential at any R. At each R, the calculated potential is the lesser of DFAP and SRP. As would be expected, the SRP is lower than the DFAP for small values of R. For large values of R the SRP cannot treat the asymptotic single-source meson clouds correctly; only an approximation based on the DFA can be used for large R. As R decreases, the difference between the T=0 potential and the T=1 potential grows; at the value R_c the difference becomes equal to the meson mass. For $R < R_c$ the lowest T=0 state is not the one that gives the T=0 SRP, but rather its energy is bounded by the energy of the lowest T=1 state plus the energy m of a meson at infinity.[13]

Figure 3 shows a plot of the combined lowest potentials. The cusp in the T=1 potential shows that the DFAP needs to be improved by using more states in the subspace in which H_{int} of Eq. (21) is diagonalized. The T=0 potential in the vicinity of R_c can be improved by adding to the subspace states with one external-meson creation operator, so that the lowest T=0 state becomes

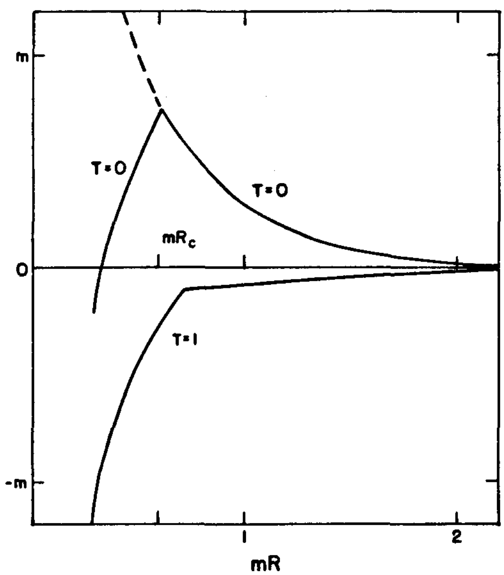

Figure 3

$$\alpha|T=0\rangle + \int f(\vec{k})\ \{a^+_{ext}(\vec{k}),|T=1\rangle\}^{T=0}\ d\vec{k} \quad ; \tag{22}$$

with this ansatz, the equations for α and $f(\vec{k})$ are like the equations for the solution of the Lee model in the V sector and are easily solved (this is not meant to imply that this has actually been done as yet). The result must be to round the cusp in the T=0 potential and to move the actual threshold for conversion of the T=0 state into the T=1 state plus a meson at infinity to a value of R smaller than R_c.

For small enough values of R both the T=0 and T=1 potentials are attractive. It is expected that this is a general feature of mesonic interactions when the source operators are nonabelian (like τ or σ). In the Abelian case it is known[13] that the full static potential is identical to the OMEP (there is no coupling-constant renormalization in this case), so that there can be a repulsive potential at short distances. The fact that nonabelian mesonic interactions always give attraction at short distances means that it is difficult to achieve saturation of nuclear forces in the usual old-fashioned way by using meson-exchange potentials with appropriate mixture of various nonabelian terms.

Summary and prospects

The techniques are now available for doing accurate computations of static potentials arising from the exchange of virtual mesons. Such computations must take account of the fact that different approximation methods must be used in the regions where R is large and where R is small. In the asymptotic region, the distorted-field approximation provides an appropriate starting-point, but it must be improved before trustworthy results are obtained for all but the largest values of R. In the region of small R, accurate strong-coupling methods are based on the use of states with coherent meson pairs. For small R, it is also important to take account of the possibility of meson emission or near-emission.

Current work is aimed at applying the techniques described here to the case of static sources interacting via pion field. In particular, it will be interesting to see how sensitive the potential is to the value of the cutoff Λ. Other areas of application are the study of the effects of nonlinearity[17] and models of quark-quark and quark-antiquark potentials.

This work was performed under the auspices of the U. S. Department of Energy.

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