An Application of Projected Distance Cross-Correlation for Abell Clusters

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Abstract

A new method is applied for the estimation of the spatial two-point correlation function of Abell clusters. This method is based on the projected distance cross-correlation function w_{SA} between the Struble and Rood catalogue and the Abell catalogue. The σ-correlation function is determined, the best fit is $w_{SA}(\sigma) = \sigma_{SA}/\sigma$, where $\sigma_{SA} = 2.3\,\mathrm{Mpc}$. The spatial cross-correlation function can be derived from $w_{SA}(\sigma)$ by an integral transformation. In our case, we obtain $\xi_{SA}(r) = (r/r_{SA})^{-\gamma}$, where $\gamma = 1.88$ and $r_{SA} = 40\,\mathrm{Mpc}$. A further normalization, based on a comparison of angular correlations, is needed to obtain the autocorrelation function since the Struble and Rood catalogue is not a fair subsample of the Abell catalogue. Thus we get $\xi_{AA}(r) \approx (r/r_{AA})^{-\gamma}$, where $\gamma = 1.88$ and $r_{AA} = 33\,\mathrm{Mpc}$. We note that the estimation is very sensitive to the estimated distance limits of the Abell catalogue. This result agrees well with $\xi_{AA}(r) = (r/30\,\mathrm{Mpc})^{-2}$ obtained from the Limber equation.

1 Motivation

The spatial correlation of Abell clusters is an important cosmological problem because the Abell catalogue (1958) is the best defined catalogue of clusters. The problem is that we do not know the spatial positions of these clusters, we have only angular coordinates for most of them. The Struble and Rood catalogue which contains Abell clusters with measured redshifts was published in 1987. This gave a chance to check the previous estimations of the spatial correlation function.

2 Data

The two catalogues were used for our calculations:

- the Abell catalogue (1958) including 2712 clusters
- the Struble and Rood catalogue (1987) including 588 clusters *with measured redshifts.*

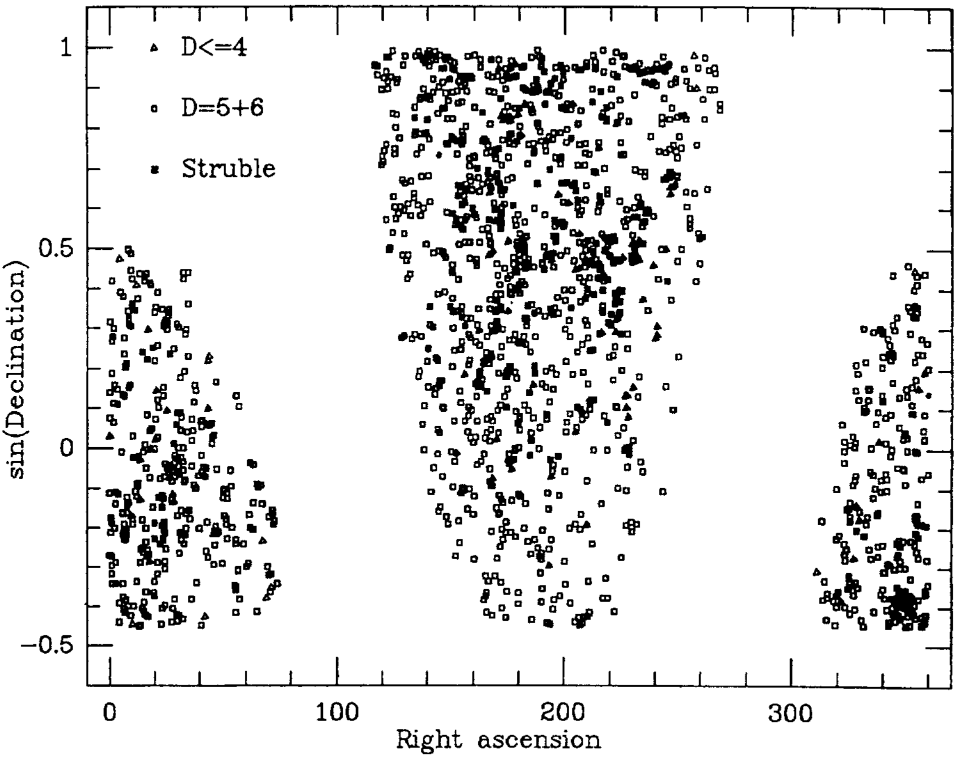

Fig. 1. The Abell catalogue and its subset, the Struble and Rood catalogue with measured redshifts

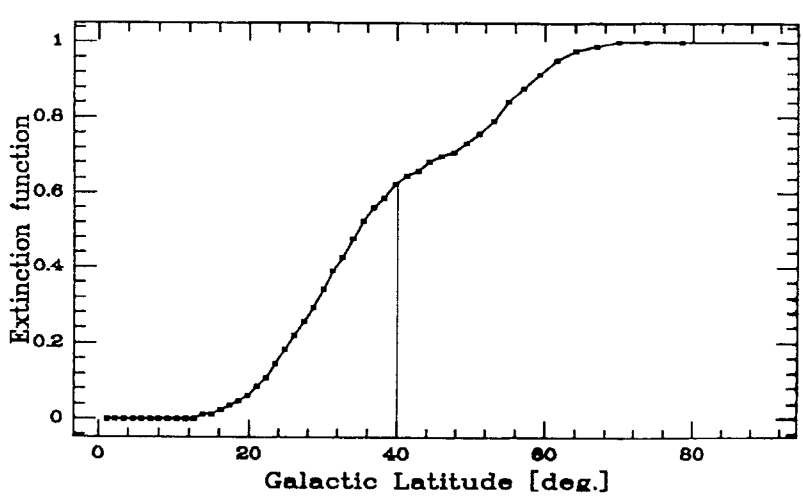

Fig. 2. Extinction function of the Abell catalogue.

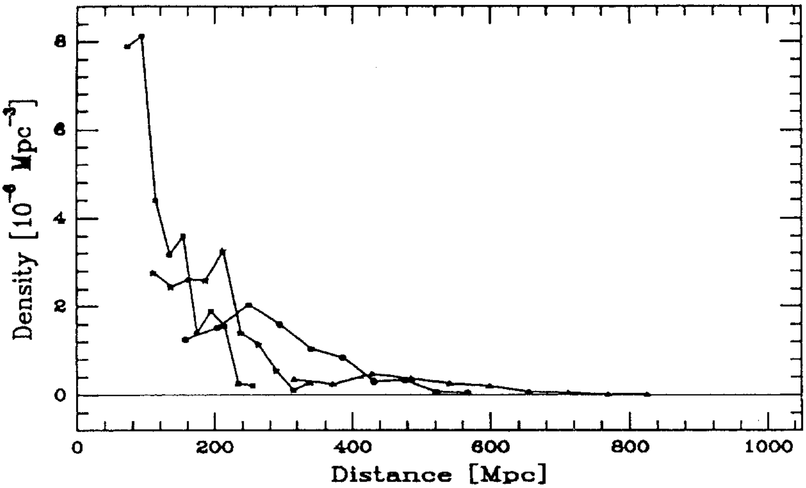

Fig. 3. Densities of the Struble and Rood clusters in various distance classes.

The cluster distributions in projected spherical coordinates are shown in *Fig. 1.* The smoothed extinction function of the catalogues, which is the effect of galactic obscuration, is shown in *Fig. 2.* (For the original extinction curves, see *Fig. 3* in Tóth *et al.*, these proceedings, p. 201). Only the high latitude samples ($|b^{II}| \geq 40°$) of the catalogues with 1418 and 310 rich clusters, respectively, were used in order to avoid regions of poor statistics. The extinction curve was also applied to the random catalogues.

3 Methods to estimate the spatial correlation

Let us summarize the previous estimation methods:

We know a nearly fair subsample of the Abell catalogue with measured redshifts. It contains all clusters in the distance classes $D \leq 4$, thus we can get their spatial correlation directly. The result can be written as a power law (Bahcall and Soneira 1983):

$$\xi(r) \approx \left(\frac{r}{r_0}\right)^{-\gamma} , \tag{1}$$

where $\gamma \approx 1.8$ and the correlation length $r_0 \approx 24$ Mpc.

We can calculate an *'approximate'* spatial correlation function by using the $m_{10} - Z$ relation as a redshift estimator.

We can derive the spatial correlation function from the angular correlation which can be estimated easily. The Limber equation, which is the connection between them, can be inverted at small angular separation by assuming a smooth selection function.

Table 1. Estimation of distance limits by Peebles and from the Struble and Rood catalogue.

	$D \leq 3$	$D = 4$	$D = 5$	$D = 6$
Seldner and Peebles [Mpc]	0 – 194	194 – 292	292 – 440	440 – 664
Struble and Rood [Mpc]	0 – 240	100 – 320	150 – 500	300 – 720
Number of clusters	38	57	565	758
Density $[10^{-6}\,\mathrm{Mpc}^{-3}]$	4.5	2.8	8.1	3.1

3.1 Distance limits of Abell's distance groups

Figures 3 and *4* show the spatial density of clusters for each Abell distance class in the Struble and Rood catalogue and the normalized densities, respectively, to check the given limits of the distance classes. It shows clearly that Abell's categorization, which was based on the $m_{10} - Z$ relation, has large errors and leads to strong overlaps.

3.2 Estimation of distance limits

In addition to the original distance limits of Abell we know an estimation by Peebles (1977), given in Table 1. It can be compared with *Fig. 4* where the measured redshifts verify them more or less. The densities of the distance classes can be computed considering these distance limits. We find that the densities are far from constant but this scatter can be explained by the uncertainities of the distance limits and the overlaps. We assume that the selection function is equal to 1 up to 600 Mpc and 0 above. From the present data we cannot derive more.

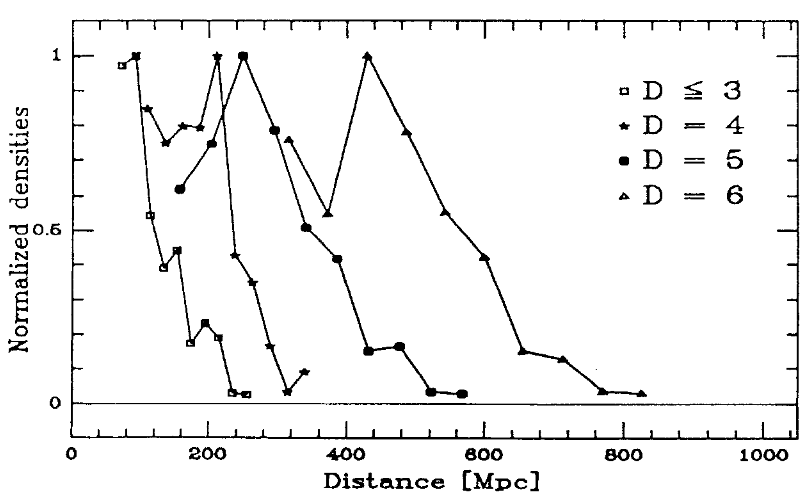

Fig. 4. The same as before, but with normalized curves, The figure shows well the distance limits of the Abell distance classes.

3.3 The projected distance cross-correlation function

Let σ denote the projected distance of two clusters. This means, that if one of them has a measured redshift and its distance is y, and the angular separation between them is θ, then $\sigma = y \cdot \theta$.

Now we define $w(\sigma)$, the projected distance cross-correlation function as follows: dn is the expected number of A-clusters in the solid angle element $d\Omega$ at separation σ from an S-cluster. The letters A and S refer to the Abell catalogue and the Struble and Rood catalogue, respectively.

$$w_{SA} = \overline{n} \cdot (1 + w_{SA}(\sigma)) \cdot d\Omega \tag{2}$$

The next formula gives the best method to estimate the σ-correlation function avoiding edge effects (the $\langle\ \ \rangle_\sigma$ symbol denotes the number of pairs whose projected distances are σ):

$$w_{SA} \approx \frac{\langle D_S D_A \rangle_\sigma}{\langle D_S R_A \rangle_\sigma} - 1 \,. \tag{3}$$

Our best power law fit,

$$w_{SA}(\sigma) = \frac{\sigma_{SA}}{\sigma} \,, \tag{4}$$

where $\sigma_{SA} = 2.3\,\mathrm{Mpc}$, is shown in *Fig. 5*.

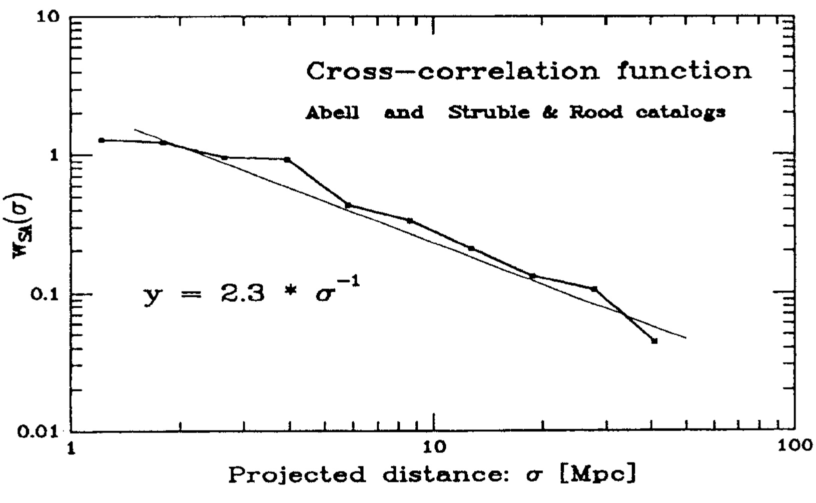

Fig. 5. Projected distance cross-correlation function between the Abell and Struble and Rood catalogues.

3.4 Relation between $w_{SA}(\sigma)$ and $\xi_{SA}(r)$

The σ-correlation function can be expressed in terms of the spatial cross-correlation function, similar to the Limber equation

$$w_{SA}(\sigma) = \frac{\int_0^\infty \int_0^\infty \rho_S \, p_S(y) \, \rho_A \, p_A(x) \, x^2 \, \xi_{SA}(r) \, dx \, dy}{\int_0^\infty \rho_S \, p_S(y) \, dy \cdot \int_0^\infty \rho_A \, p_A(x) \, x^2 \, dx} , \tag{5}$$

where $r^2 = x^2 + y^2 - 2xy \cdot \cos(\sigma/y)$ (*Fig. 6*), ρ_S and ρ_A are the mean densities of clusters, p_S and p_A are the selection functions. The formula is too complicated for inversion, so we use the following approximation:

$$r^2 \approx (y-x)^2 + \frac{x}{y} \cdot \sigma^2 . \tag{6}$$

3.5 The inverse equation

The inverse equation was derived by Lilje and Efstathiou (1988). At small separation,

$$\xi_{SA}(r) = -\frac{1}{\pi B r} \cdot \frac{d}{dr} \int_r^\infty \frac{\sigma \, w_{SA}(\sigma)}{(\sigma^2 - r^2)^{\frac{1}{2}}} \, d\sigma . \tag{7}$$

The constant B is determined by the redshift distribution of the Struble and Rood catalogue and the selection function of the complete Abell catalogue:

$$B = \frac{\int_0^\infty p_A(x) \, p_S(x) \, x^2 \, dx}{\int_0^\infty p_S(y) \, dy \cdot \int_0^\infty p_A(x) \, x^2 \, dx} = 7.4 \cdot 10^{-4} . \tag{8}$$

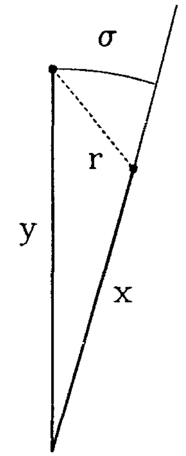

Fig. 6. Relation between the spatial and projected distance cross-correlations.

The result of the integral-transformation is

$$\xi_{SA}(r) \approx \left(\frac{r}{r_{SA}}\right)^{-\gamma}, \tag{9}$$

where $\gamma = 1.88$ and $r_{SA} = 40$ Mpc (*Fig. 7*).

Note that this is the spatial cross-correlation function and not the auto-correlation.

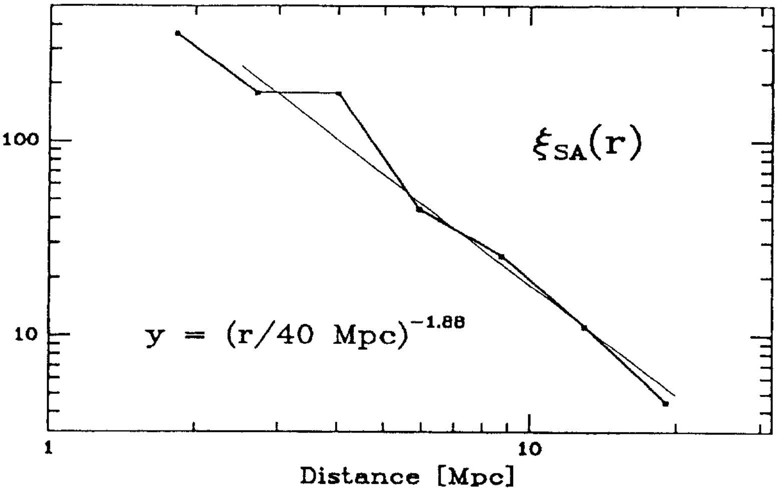

Fig. 7. Spatial cross-correlation function between the Abell and Struble and Rood catalogues.

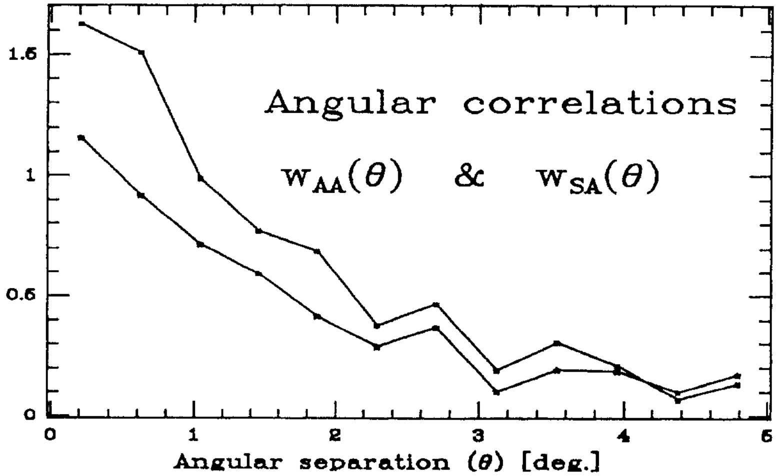

Fig. 8. Comparison of the angular correlations of the Abell and Struble and Rood catalogues.

3.6 The spatial cross-correlation function

A special normalization is needed for the spatial cross-correlation function because the Struble and Rood catalogue is not a fair subsample of the whole Abell catalogue. This means that the picking method of Struble and Rood clusters may have some virtual correlation effects which we should avoid. That effect can be seen in *Fig. 8* which shows the angular auto- and cross-correlation functions.

We found that their ratio is nearly constant, ≈ 1.45 up to 4 degrees (*Fig. 9*). Accepting the simple assumption that the ratios of the angular and spatial correlation

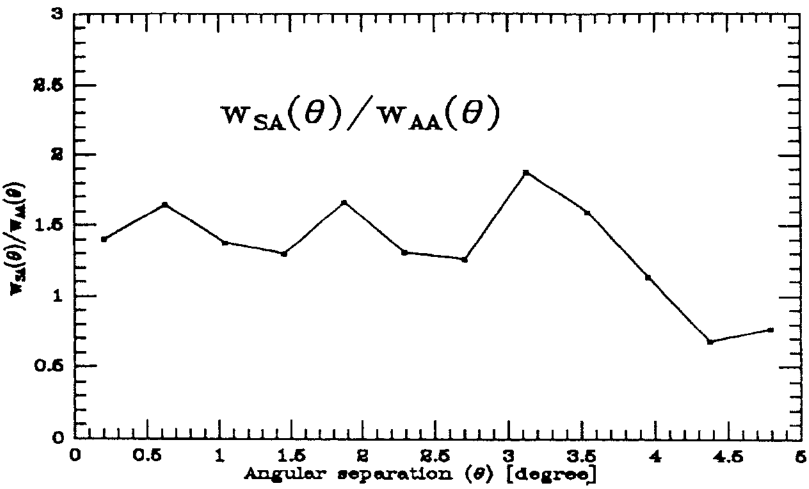

Fig. 9. Ratio of the angular correlation functions of the Abell and Struble and Rood catalogues.

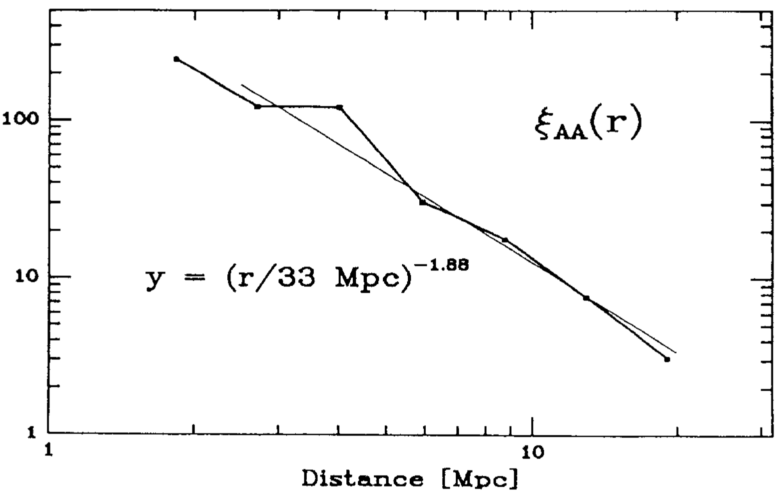

Fig. 10. Spatial correlation function of the Abell clusters.

functions are about the same, we can correct the spatial cross-correlation function to the autocorrelation function of the Abell clusters. This assumption can be derived from the fact that connections of correlation functions are linear.

4 Conclusion

Our best power law fit to the spatial correlation function of the whole Abell catalogue is the following (*Fig. 10*):

$$\xi_{AA}(r) \approx \left(\frac{r}{r_{AA}}\right)^{-\gamma} , \tag{10}$$

where $\gamma = 1.88$ and $r_{AA} = 33$ Mpc. We note that the amplitude depends on the distance limit of the Abell catalogue. The dependence has the following form:

$$r_{AA} \propto D^{\frac{3}{\gamma}} , \tag{11}$$

thus from the $\approx 10\,\%$ uncertainty of D:

$$r_{AA} = 33 \pm 5\,\mathrm{Mpc} . \tag{12}$$

Our result is in a good agreement with the inversion of the Limber equation. Although this estimation gives a somewhat stronger correlation than the previous estimation of Bahcall and Soneira based on the nearest 104 clusters, the exponent is nearly the same.

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