# Galaxy Redshift–Number Counts with MRSP Data: A Method of Estimating $q_0$

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**Abstract**

An $N(M,z)$ test, using 6 300 galaxy redshifts from the MRSP, leads to a statistical determination of $q_0$. While the current range of $0 < q_0 < 0.25$ is only preliminary, the method seems promising when applied to a forthcoming larger sample of galaxy redshifts.

## 1 Introduction

The Muenster Redshift Project (MRSP) combines medium redshift range, large coverage of solid angle across the sky and sufficient number density of galaxies to permit us to apply the galaxy $N(M,z)$ test of observational cosmology. The redshift–number test, used here in bins of *absolute magnitude*, is more sensitive to the geometry of the universe than the more familiar flux–number test and less affected by galaxy evolution (Weinberg 1972, Loh and Spillar 1986, Sandage 1987). In *physical* terms, it is a redshift–volume test, sensitive to all kinds of gravitating matter (Loh 1988).

## 2 Theoretical framework

In standard Friedmann models **differential number counts** in given intervals of absolute magnitude $M$ and redshift $z$ are given by

$$N(M,z) = dM\, d\omega\, dz\, \mathrm{LF}(M,z)\, g(z,H_0,q_0)\ . \tag{1}$$

$d\omega$ is the solid angle covered; in the case of no evolution and of galaxy number conservation in a comoving volume

$$\mathrm{LF}(M,z) = \mathrm{LF}(M,0)\ (1+z)^3\ , \tag{2}$$

where LF(M, 0) is the present galaxy luminosity function; the term $g$ describes the geometry, here of the matter dominated universe:

$$g = \left(\frac{c}{H_0}\right)^3 \frac{1}{q_0^4} \frac{\left(zq_0 + (q_0-1)\left(\sqrt{2q_0z+1}-1\right)\right)^2}{(1+z)^6\sqrt{2q_0z+1}}\ . \tag{3}$$

All equations are taken from Weinberg (1972).

The number counts open the possibility of estimating $q_0$ when all other quantities are known or can be properly estimated. In particular, our method does not require an explicit expression for the LF; instead we only assume its *invariance* over a certain limited redshift range.

## 3 The galaxy data

A homogeneous (sub)sample of the MRSP data is used, obtained from the combined UKST objective prism and direct plates of the ESO/SRC Atlas field No. 411. It contains 6 300 galaxies with measured redshifts $z < 0.3\,(\pm 0.008\,m.e.)$ and apparent magnitudes $16\overset{m}{.}5 < m_B < 20\overset{m}{.}5\,(\pm 0.1\,m.e.)$, distributed over a solid angle of $30\,\mathrm{deg}^2$. *Fig. 1* is the $M_B$ *vs.* $z$ diagram, showing all measured galaxies. Two strips for counting are included, one of width 0.5 in $M$ extending over all $z$ values, one of width 0.01 in $z$, extending over all $M$ values. The data show a sharp cutoff at the limiting magnitude of the survey and a stochastic dying out of numbers for bright galaxies. The latter is due to a real decrease in density but also to the upper brightness limit of the $z$ measuring method. For technical details of the measurements and reductions of the raw data, see Horstmann (1988) and Schuecker (1988a).

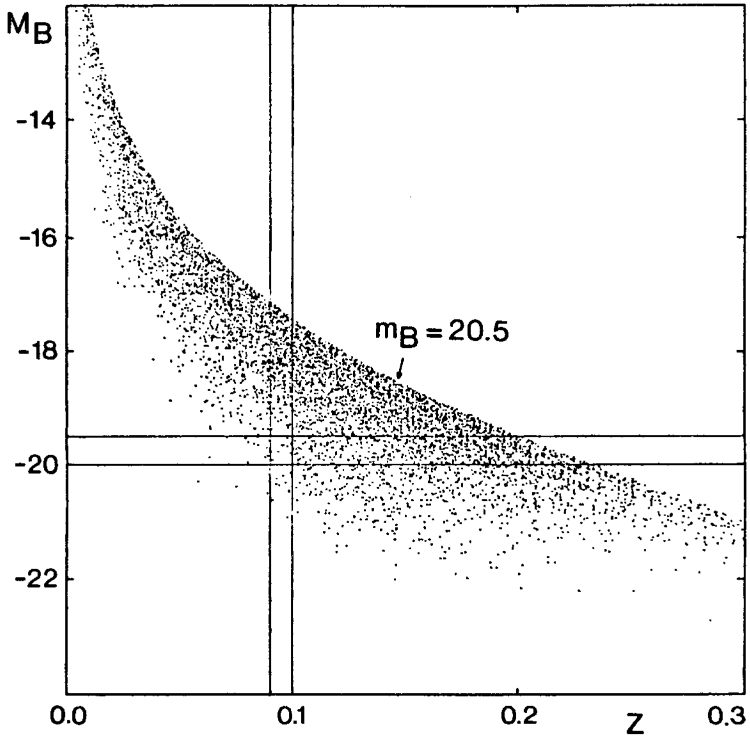


**Fig. 1.** Absolute magnitudes $M_B$ *vs.* redshift $z$ for all galaxies investigated. Indicated are two counting strips of widths $\Delta M = 0.5$ and $\Delta z = 0.01$. The data range is limited by faint and bright apparent magnitudes.

## 4 Data corrections

Several systematic effects bias the *empirical* galaxy counts $n(M, z)$. The most important ones depend on apparent brightness.

1) Due to the decreasing likelihood of including a galaxy into the counts near the bright as well as near the faint magnitude limit, *edge effects* are found at both low and high $z$ values; $n_M(z) := n\,(M = const, z)$ and $n_z(M) := n\,(z = const, M)$ become too small – the curve thins out at both ends. As a simple but effective correction we use appropriate cuts.

2) The $n_M(z)$ distributions suffer from a slowly changing effectivity of $z$ measurability with apparent brightness, as shown by Schuecker (1988b, his *Fig. 1a, b*). A linear correction function $s(m)$ is applied, obtained from the ratio of galaxy numbers on the direct plate, which are not affected by this bias, and the numbers on the objective prism plate. An updated version of Schuecker's redshift measuring method is noticeably less influenced by this effect so that for future data no corrections are needed.

3) Due to observational errors in redshift and magnitude there is a net *migration* of galaxies across the $M, z$-plane, affecting the counts in the bins. The observed galaxy distribution is a convolution of the real underlying distribution with the observational errors. For steep gradients in the counts the bins are systematically *filled up* in the direction of decline and *emptied* in the direction of increasing numbers. Corrections depend on the affected parameter

- $M$ : If the LF does not vary too much over the redshift range, there should be no pronounced effects in the counts $n_M(z)$; the *effective* LF remains the same and does not require correction.
- $z$ : Numerical simulations (convolution of assumed underlying distributions with redshift errors) show major effects at the beginning and the end of the $N_M(z)$ distributions. We apply a correction by cutting both ends appropriately. Also, the selection procedure described in Sect. 5 helps to avoid this bias.

4) Another bias, which depends on the adopted world model, will be introduced through the calculation of absolute magnitudes. A value for the deceleration parameter $q_0$ must be assumed in order to determine $M$. The influence of *false* adopted values has been estimated theoretically: By using the luminosity distance given by Mattig (1958), we found maximum systematic shifts $\Delta M(\max) < 0.2$ in the range $0.03 < z < 0.2$ and $0 < q_0 < 1$. This is of the order of the uncertainty in the absolute magnitudes. It has also been estimated empirically: By assuming values for $q_0$ of 0.1, 0.5, and 1.0 with the present data set, slight systematic changes in the corresponding $n_M(z)$ curves, but no significant changes of *slope*, the relevant parameter in this context, were found.

In addition to these more technical problems, a *physical* bias in the $n_M(z)$ counts may arise from the fact that evolution is not entirely negligible, even in a $z$-interval as small as 0.2, and from real density fluctuations (clusters of galaxies, voids) which change the count distributions. A possible "correction" for fluctuations is filtering to obtain a smoothed distribution. This, however, touches on the fundamental problem of how to deal with large inhomogeneities. Are we allowed to smooth, cut, neglect them as we like?

Here, we chose to ignore all possible perturbations of this kind. No corrections are needed for biases in *absolute* number counts, because only the slope of the $n_M(z)$ curves, *i.e.* *relative* numbers are important in the following analysis.

## 5 Analysis of the data

Because the MRSP is a magnitude-limited survey whose degree of completeness remains constant within certain limits of *apparent* magnitude, galaxies of different *absolute* magnitudes show different coverage of the redshift ranges and corresponding counts cannot simply be added without introducing an appreciable bias.

In order to be able to combine galaxies in all magnitude strips we adopted a **normalization** procedure consisting of three steps:

1. **Division** of empirical counts $n_M(z)$ obtained in regions overlapping in $z$ and adjacent in $M$ (*$z$-overlapping, $M$-adjacent*), without an explicit assumption about the LF.

2. **Selection** of the most *coherent* parts from the distribution of ratios obtained in 1), assuming invariant (or covariant) LFs with redshift.

3. **Normalization** of all counts to an *arbitrary constant* value $\text{LF}_0$, using the selected mean ratios obtained from 2).

Comments:

*ad* 1)

We expect

$$\frac{N(M_i,z)}{N(M_j,z)} = \frac{\text{LF}(M_i,z)}{\text{LF}(M_j,z)} =: K_{ij}(z) \tag{4}$$

with $K_{ij}(z) = \text{const}$ for invariant LF(z) or covariance of $\text{LF}(M_i)$ and $\text{LF}(M_j)$ with redshift. We then expect the empirical values

$$k_{ij}(z) := \frac{n_{M_i}(z)}{n_{M_j}(z)} \tag{5}$$

to be constant within stochastic fluctuations, when the actual LFs do not change with redshift or when they change in the same sense, and when no systematic bias affects the counts.

*Figure 2* shows a test of the expectation $k_{ij}(z) = \text{const}$. Four ratios $k_{ij}(z)$ of *$z$-overlapping, $M$-adjacent* counts *vs.* redshift are shown. Adjacency is not a necessary condition, but it provides the largest overlapping regions. The coherence requirement helps to avoid an arbitrary selection of data points.

We find high noise (stochastic and/or systematic) at the beginning and at the end of the distributions and fairly flat central parts. Slight systematic shifts are present (differential density fluctuations between the curves or different shapes of LFs?), but the small number of data points does not permit conclusions. In order to avoid the noise we use cuts at both ends.

*ad* 2)

Selection of the coherent parts in the curves of *Fig. 2* is presently performed interactively and thus somewhat arbitrary. In the future statistical selection criteria will be used automatically.

*ad* 3)

Using the selected ratios $k_{ij}(z)$ we derive mean conversion factors for all pairs of *z-overlapping, M-adjacent* counts. They depend only on the assumption of invariant (or covariant) LFs with redshift, which is justified by the present distribution of the ratios.

The counts are normalized to an arbitrary value $LF_0$ after the mean conversion factors have been applied, leading to $n_0(z)$ and, averaged over all counts within a given red-

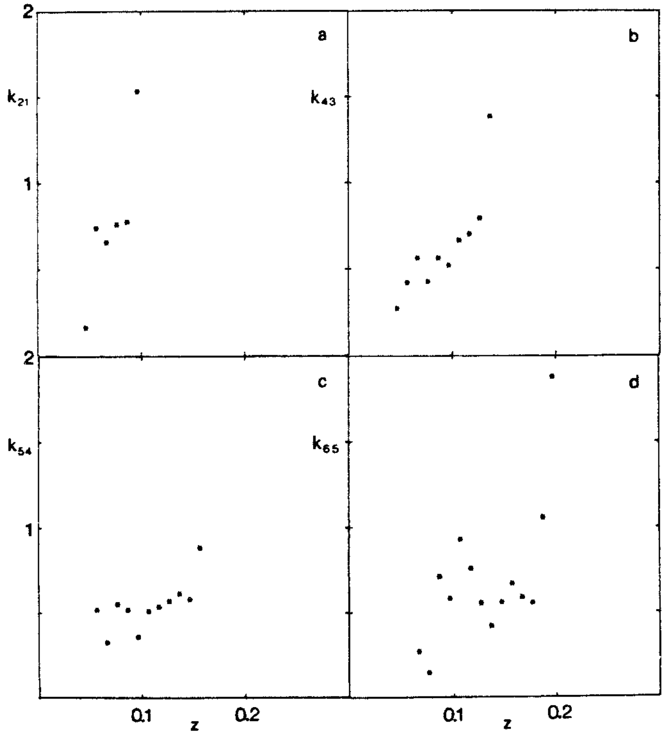


**Fig. 2.** Ratios $k_{ij}(z)$ from four *z-overlapping, M-adjacent* counts *vs.* redshift (definitions see text). The expectation $k_{ij}(z)$ = const, indicating an invariant (or covariant) LF(z) for pairs is a fair assumption for the central parts of the point distributions.

shift interval, to $\bar{n}_0(z)$. The normalized counts do not contain any information about the LF, but fully conserve their dependence upon the universal geometry $g(z, H_0, q_0)$, in which we are interested.

## 6 Results

The normalized counts are tracers of the universal geometry and can be compared with the theoretically expected values

$$N(M, z) = A \cdot N(z, q_0) = A \cdot g \cdot \left(\frac{H_0(1+z)}{c}\right)^3 , \tag{6}$$

where $A := d\omega\, dz\, c^3\, H_0^{-3}\, \mathrm{LF}_0$ includes all parameters assumed to be constant and $g$ represents the global geometry.

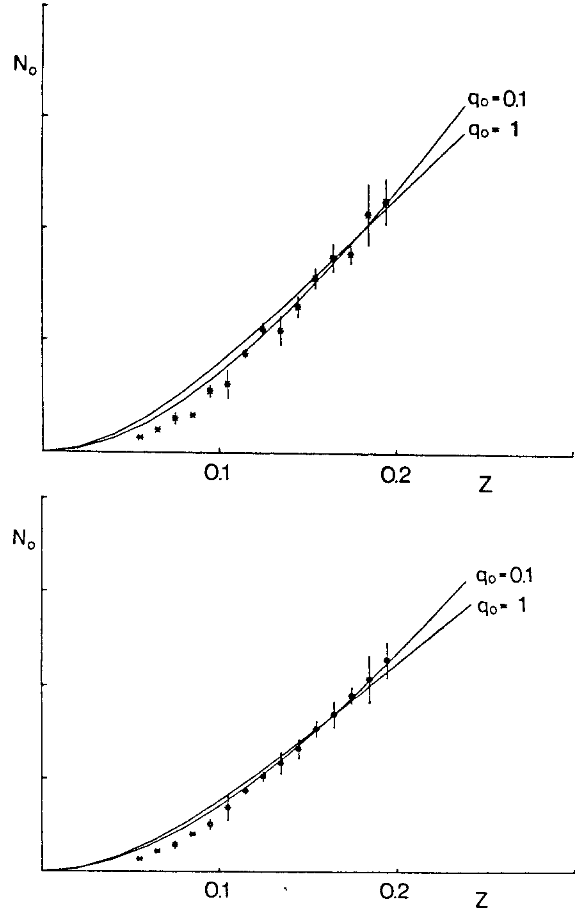


**Fig. 3.** Normalized mean number counts *vs.* redshift and two best-fit theoretical lines for $q_0 = 0.1$ and $q_0 = 1$ through the weighted points with $z > 0.1$.
3a: original distribution of data points; 3b: smoothed distribution of data points.

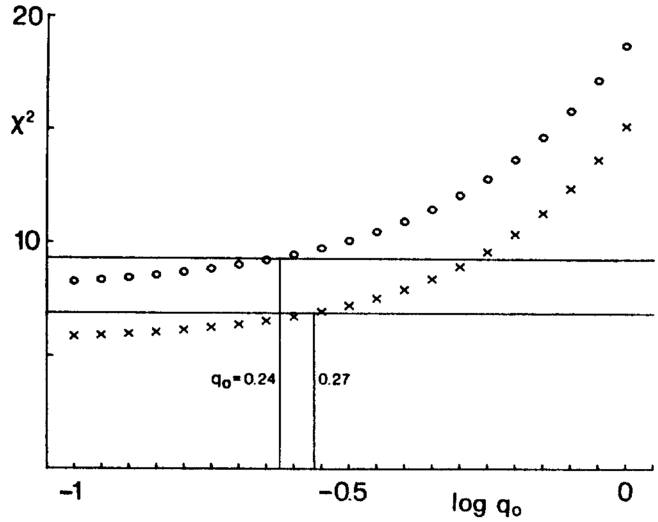


**Fig. 4.** $\chi^2$ *vs.* $\log(q_0)$ for weighted (circles) and unweighted (crosses) fits through all points of *Fig. 3* with $z > 0.1$. Indicated are the upper $1\sigma$ limits. The minima of $q_0$ lie too close to zero to be represented. The fits to the weighted data define a smaller $q_0$ range and show $\chi^2$ values consistent with the statistical expectation.

The comparison is made using least squares fits, weighted and unweighted, of the relation

$$\bar{n}_0(z) = A \cdot N(z, q_0) \; . \tag{7}$$

The assumption of Poisson statistics attributes to each count $n$ the weight $p(n) = (\text{var}\,(n))^{-1} = n^{-1}$.

*Figure 3a* shows the normalized mean number counts *vs.* redshift and two best-fit theoretical lines for $q_0 = 0.1$ and $q_0 = 1$, respectively, through all weighted points in the most reliable range $0.1 < z < 0.2$. The indicated error bars are obtained from Poisson statistics of the actual numbers involved. The fit for $q_0 = 0.1$ is better than for $q_0 = 1$. The difference of the two fits appears more conspicuous, when a block filter is applied for smoothing the empirical counts (*Fig. 3b*).

In *Fig. 4* the $\chi^2$-values of the fits as functions of the free parameter $q_0$ for the weighted and unweighted data are shown. The best-fit values for $q_0$ and the lower $1\,\sigma$ limits are not apparent because both lie near zero. The upper limits are marked in the figure. The weighted fits give better (not smaller!) $\chi^2$ values, confirming with the value 1.04 the expectation of $\chi^2 = 1$ per degree of freedom and thus justifying our weighting procedure.

As the result we adopt the $1\sigma$ range $0 < q_0 < 0.25$ for the deceleration parameter. This represents a very preliminary value, following from galaxy counts of *one* Schmidt plate only. With much more data (recently 24 000 redshift measurements have been completed) it seems not unrealistic to expect a better estimate of $q_0$ in the near future.

## References

Horstmann, H., 1988. *These proceedings*, p. 111.

Loh, E.D., 1988. *Astrophys. J.*, **239**, in press; see also *these proceedings*, p. 267.

Loh, E.D., Spillar, E.J., 1986. *Astrophys. J.*, **307**, L1.

Mattig, W., 1958. *Astr. Nachr.*, **284**, 109.

Sandage, A., 1987. In *IAU Symp. 124, Observational Cosmology*, eds. Hewitt, A., Burbidge, G., Li, Z.F., Reidel, Dordrecht, p. 1.

Schuecker, P., 1988a. *These proceedings.*, p. 142.

Schuecker, P., 1988b. *These proceedings.*, p. 160.

Weinberg, S., 1972. *Gravitation and Cosmology*, Wiley, New York.