THE ELECTRIC RESISTIVITY OF A MAGNETIC SEMICONDUCTOR WITH EASY-AXIS OF ANISOTROPY POPULATED BY MAGNON SOLITONS

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1. INTRODUCTION

We examine the possibilities of the formation of solitonlike magnon excitations in "quasi-one-dimensional (QOD) ferromagnets with "easy-axis" anisotropy (EAA). Such a possibility is indicated[1] under the constraint that the magnetic anisotropy energy is small compared to the direct exchange energy between magnetic ions. It seems us that good candidates which can support the existence of such solitons may be found among ferromagnetic semiconductors (FMS), first of all europium oxide (EuO).

Not a single FMS had been discovered until 1960; moreover an opinion had been voiced that ferromagnetic and semiconducting properties were incompatible. After discovering first FMS ($CsBr_3$) their number was growing and at present is close to 100. Nagaev[2] states the data showing that, for example EuO crystal possesses properties which allow it to be mapped approximately onto EAA chain. The EuO crystals have cubic, of NaCl type, structure. Their importante feature is the absence of an orbital angular momentum of the electrons in the partially filled f-shells of the Eu^{++} ions. The ground state of those f-shells is the $^8S_{7/2}$ with L=0, S=7/2. The magnetic dipole--dipole energy of ferromagnetically ordered spins in that cubic lattice is zero. Those are the reasons for the crystallographic anisotropy of such crystals to be relatively very small (the anisotropy field for EuO is equal to 0,02 T while the effective exchange field is of the order of 10^2T). On the other hand, such crystals appear to be almost ideal Heisenberg magnets[2].

The second, and main stage of our work is dedicated to examination of collisions of conduction electrons with magnon solitons in a FMS with EAA. It is well known that the existence of localized spin moments that couple to the conduction electrons (CE) has important consequences on the electrical conductivity of the corresponding crystals. The magnetic ions act as scattering centers so that at sufficiently low temperatures the scattering they cause will be the primary source of electrical resistance (ER).

Vonsovsky[3] first realized that an important contribution to ER could occur in ferromagnets as a result of the exchange interaction among CEs and the localized magnetic ions (LMI), often called the s-d or s-f interaction. Nagaev emphasizes[2] that the model proposed by Vonsovsky represents the most adequate description of FMSs. In his model the electrons in LMI's d or f shells interact with one another via the Heisenberg nearest-neighbor exchange mechanism while an entirely distinct subsystem exists which is composed of quasifree CEs in Bloch states of the conduction band (s). Since in this model the localized d and f electrons can be analized using a virtually identical treatment we will use the symbol "l" (localized) for both.

2. THE VONSOVSKY MODEL

Let us construct the s-l model Hamiltonian due to Vonsovsky

$$H_V = H_s + H_l + H_{sl} \ . \tag{1}$$

The first term on the right-hand side represents the noninteracting CEs described by the operators $a_{k\sigma}$, $a^+_{k\sigma}$ which destroy and create an electron with the wave number k and with up (↑) and down (↓) spin projection ($\sigma=\pm 1/2$), respectively

$$H_s = \sum_{k,\sigma} E_{k\sigma} a^+_{k\sigma} a_{k\sigma} \ . \tag{2}$$

The normalized energy of CEs, induding the Zeman energy due to an applied external magnetic field h (AEMF), and the zone shift caused by s-l interaction has the form

$$E_{k\sigma} = \frac{\hbar^2 k^2}{2m^*} + \sigma g_s m_B h - S W_{kk} \delta_{\sigma\downarrow} \tag{3}$$

where m* represents the effective mass of a CE; g_s is the Lande factor; m_B is the Bohr magneton; S is the spin of a single LMI; W_{kk} is the interaction energy of Vonsovsky type which arises from thirth term on the right hand side of Eq. (1).

The compounds of europium may serve as an example with a wide conduction band. In them the effective mass of CEs is of the order of free-electron mass m_0. On the basis of this, and taking into account that the lattice constant is of the order $R_0 \simeq 3\text{-}5 \cdot 10^{-10}$ m, we may estimate that the bandwidth A in Eu chalogenides is of the order $A \simeq 3\text{-}5$ eV.

The second term on the right-hand side of Eq. (1) represents the interionic magnetic interaction which is of direct or indirect exchange type

$$H_l = -g_s m_B h \sum_n S^z_n - \frac{J_0}{4} \sum_n (S^+_n S^-_{n+1} + S^-_n S^+_{n+1}) - \frac{J^z_0}{2} \sum_n S^z_n S^z_{n+1} \ , \tag{4}$$

where the spin ladder operators are $S^{\pm}_n = S^x_n \pm i S^y_n$ and the spin projection operator in the directon of AEMF is S^z_n. The energy parameters introduced above are the exchange interaction constant J_0 and anisotropic exchange interaction constant J^z_0.

The last term on the right-hand side of Eq. (1) describes the exchange coupling between the CEs and the spins of LMIs. In the single-band approximation this can be expressed in the direct space representation as follows

$$H_{s-1} = -\sum_{n,\sigma,\sigma'} W_{n\sigma\sigma'}(S_n \cdot s)_{\sigma\sigma'} a^+_{n\sigma'} a_{n\sigma} , \tag{5}$$

where the exchange energy $W_{n\sigma\sigma'}$ is of short-range type; $s_{\sigma\sigma'}$ are Pauli matrices, and $a^+_{n\sigma}$, $a_{n\sigma}$ are Fourier transforms of the operators a^+_k, a_k. The EuO has rather strong constant of interaction of the order $W \sim 0{,}1eV$.

3. THE SOLITONLIKE BOUND STATE OF N MAGNONS

The formation of bound states of a few magnons in QOD ferromagnet with EAA was experimentally observed for the first time by Torrance and Tinkham[4]. The basic theoretical discussion about necessary conditions for the formation of such bound states in the same crystals is given by Ivanov and Kosevich[1]. The starting point in this discussion is the assumption about smallness of anisotropy, i.e. the relative magnitude of the energy $J^z_0 - J_0$ in comparison with the exchange energy J_0. For EuO this condition is fairly fulfiled due to $(J^z_0 - J_0)J_0^{-1} \simeq 10^{-3}$.

We used a quantum-mechanical method formulated in the space of coherent states[5] and our main results are congruent with those obtained in Ref. 1 in terms of a classical approach.

We start from Hamiltonian (4). The Holstein-Primakoff representation, which is adequate description due to $S=7/2$, allows us to go over from the spin operators to the magnon annihilation and creation operators, B_q, B^+_q. In such a way obtained Hamiltonian of the "magnon gas" in the k representation is given by the approximate expression

$$H_1 = \sum_q \epsilon_q B^+_q B_q - N^{-1/2}(J^z_0 - J_0) \sum_{q,q_1,q_2} B^+_{q+q_1} B^+_{q_2-q_1} B_q B_{q_2} \tag{6}$$

The energy of a one magnon state is defined in terms of the corresponding dispersion relation

$$\epsilon_q = g_s m_B h + 2SJ^z_0 - 2J_0 S\cos(R_0 q) . \tag{7}$$

The second term on the right-hand side of Eq. (6) represents the magnon-magnon attractive interaction. Even in the long-wave limit ($R_0 q \to 0$) the mentioned interaction remains nonzero as a result of a non-zero spin-wave collision amplitude.

We formed the ansatz trial function as a product of Glauber's coherent states and after straightforward procedure by virtue of usuage the continuum approximation, Schrödinger equation with cubic nonlinearity (NLSE) appears.

Assuming that a fixed number of magnons N is involved in clusterization in terms of attractive magnon-magnon forces, we solved NLSE including the normalization condition $\int_{-\infty}^{+\infty} |\psi(z,t)|^2 dz = N$. The corresponding envelope of the clusterized "magnon drop" moves along the chain with velocity v in the form of a bell shaped soliton. After performing the Fourier transform of the bell shaped solution $\psi(z,t)$ we express the density of magnons with the wave vector q involved in a cluster as follows

$$<N_q> = \frac{dN}{dq} = |\psi_\varepsilon(t)|^2 = N \frac{\pi^2}{2\mu} \operatorname{sech}^2\{\frac{\pi R_o}{2\mu}(k_s - q)\} \ . \tag{8}$$

The corresponding parameters have the following meanings: k_s is the solitonic quasi-wave number; μ represents the inverse solitonic domain of localization and $\hbar\omega_s$ is the energy of a cluster.

$$k_s = N \frac{\hbar v}{2J_o S R_o^2} \ ; \ \mu = N \frac{J_o^z - J_o}{2J_o S} \ ; \ \hbar\omega_s = E_o + \frac{1}{2} M^* v^2 \ . \tag{9}$$

Where the static energy of a cluster E_o and its effective mass M^* are expressed by

$$E_o = N\varepsilon_o\{1 - \frac{N^2(J_o^z - J_o)^2}{12 J_o S \varepsilon_o}\} \ ; \ \varepsilon_o = \varepsilon_{q=o} \ ; \ M^* = N \frac{\hbar^2}{2SJ_o R_o^2} \ . \tag{10}$$

The expression (8) has basic importance because it will play the role of magnon population in the expressions containing the corresponding correlators in the theory which follows in the next section.

4. THE SCATTERING PROCESSES BETWEEN CEs AND MAGNON SOLITONS AND THEIR INFLUENCE ON FMS's CONDUCTIVITY

The method is based on the formulation of nonequilibrium density matrix first proposed by Zubarev[6]

$$\hat{\rho} = \{1 + \int_0^1 d\tau \{\exp(-\hat{M}\tau)\delta\hat{M} \exp(\hat{M}\tau)\}\}\hat{\rho}_{eq} \tag{11}$$

where operator $\hat{M}$ is proportional to the diagonal parts of Hamiltonian (2) and (3) including the thermodinamical forces

$$\hat{M} = \sum_q \gamma_q B_q^+ B_q + \sum_{k\sigma} \tilde{\gamma}_{k\sigma} a_{k\sigma}^+ a_{k\sigma} \ ; \ \gamma_q = \beta_2(\varepsilon_q - \mu_2) \ ; \ \tilde{\gamma}_{k\sigma} = \beta_1(E_{k\sigma} - \mu_1) \tag{12}$$

where μ_1 is the Fermi level of the conduction band, while μ_2 is the chemical potential of the magnon subsystem; τ is dimensionless parameter. The important assumption is that in a nonequilibrium state where the magnon population is generated by an external alternative field the subsystems are at slightly different temperatures[7]

$\beta_1=(k_BT_1)^{-1}\neq\beta_2=(k_BT_2)^{-1}$; T_1 correspones to the free electron gas while T_2 characterizes the magnon's gas. Operator $\delta\hat{M}$ in Eq. (11) involves the Vonsovsky interaction between subsystems

$$\delta\hat{M} = (\beta_2-\beta_1)\int_{-\infty}^{0} \exp(\theta t)\dot{H}_s(t)dt \; ; \; \dot{H}_s = \frac{1}{i\hbar}\left[H_s,H_v\right] \; ; \; \theta\ll 1 \; . \tag{13}$$

At last, $\hat{\rho}_{eq}$ represents the equilibrium statistical operator (NSO)

$$\hat{\rho}_{eq} = \exp(-\hat{M})\{Tr\{\exp(-\hat{M})\}\}^{-1} \; . \tag{14}$$

The main step in the procedure is to find the average value of the energy current between subsystems taking the average of expression (13) with respect to NSO

$$<\dot{H}_s> = Tr(\rho\dot{H}_s) = (\beta_1-\beta_2)L_{12} \tag{15}$$

where L_{12} represents the kinetic coefficient of the scattering process

$$L_{12} = \int_{-\infty}^{0} dt\exp(\theta t)\int_0^1 d\tau\{Tr\{\dot{H}_s(0)\exp(-\hat{M}\tau)\dot{H}_s(t)\exp(\hat{M}\tau)\hat{\rho}_{eq}\}\} \; . \tag{16}$$

Performing very cumbersome calculations with the simplifications enabled by the estimations of different parameters, and averaging the kinetic coefficient over all alowed velocities of "ideal gas" of solitons (IGS) we finally get the mean relaxation time for collisions of conducting electrons with IGS as a function of constant AEMF as follows

$$<\tau(h)> = \frac{<H_s^2>}{<L_{12}>} = \frac{a(\beta_1)+b(\beta_1)h+c(\beta_1)h^2}{m(\beta_1,\beta_2,\Delta T)+n(\beta_1,\Delta T)h} \tag{17}$$

where we used the symbols as follows

$$a(\beta_1) = f_1(\beta_1)+SWf_2(\beta_1)+4S^2W^2f_3(\beta_1)$$

$$b(\beta_1) = \frac{1}{4}\, g_s m_B f_2(\beta_1)+2SWg_s m_B f_3(\beta_1) \; ; \; C(\beta_1) = \frac{1}{4}\, g_s^2 m_B^2 f_3(\beta_s)$$

$$f_1(\beta_1) = \frac{3}{8}\frac{\eta}{\beta_1^2} \; ; \; f_2(\beta_1) = \frac{1}{4}\frac{\eta}{\beta_1} \; ; \; f_3(\beta_1) = \frac{1}{8}\,\eta \; ; \; \eta = \frac{R_o}{\sqrt{\pi}}\left(\frac{2m^*}{\beta_1\hbar^2}\right)^{1/2} \tag{18}$$

$$m(\beta_1,\beta_2\Delta T) = D\left(\frac{\hbar^4 N^2}{8J_o^2S^2R_o^4m^*M^*\beta_2} - SW\right) \; ; \; n(\beta_1,\Delta T) = \frac{1}{2}\, g_s m_B D$$

$$D = \frac{\pi^2SW^2R_oNk_BT_1^2}{\mu\mu_1\Delta T\hbar^2}\left(\frac{m^*M^*\beta_2}{\pi^2\beta_1}\right)^{1/2} \times \mathrm{erf}\left(\frac{\pi^2\hbar^2\beta_1}{8m^*R_o^2}\right) \; ; \; \Delta T = T_2-T_1\ll T_2 \; .$$

We make a semiquantitative estimation of the relaxation time by using the following set of available parameters: $R_o\sim 3\cdot 10^{-10}$ m; $m^*\sim 10^{-30}$ kg, $T_1=10$ K, $\Delta T\sim 1$K; $S=7/2$, $W\sim 10^{-21}$J, $M^*\sim 10^{-30}$ kg, $J_o\sim 10^{-20}$ J, $N\sim 5$, $\mu_1\sim\mu_2\sim 10^{-19}$ J, $\mu\sim 10^{-3}$.

On the basis of aforementioned estimations we obtain the following AEMF dependence of relaxation time

$$<\tau(h)> \simeq \frac{1}{4} \left(\frac{6+3h+h^2}{1+h}\right) \cdot 10^{-8} \text{ s} \tag{19}$$

which yealds that for $h \simeq 0,7$ T the minimum relaxation time has the value $<\tau>_{min} \sim 1 \cdot 10^{-8}$s, Fig. (1). The possible conclusions could be as follows. Supppose that by the method of parametric resonance by an alternating magnetic field[8] at radio frequencies the

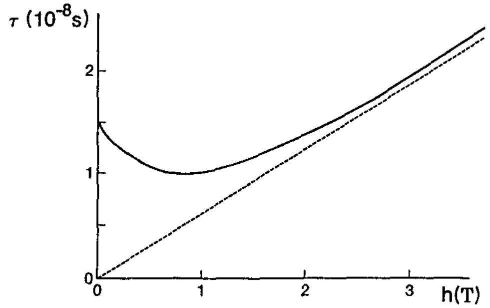

FIG. 1. The field dependence of the relaxation time corresponding to the conduction electron IGS at T_1=10 K and $\Delta T \sim 1$ K.

generation of coherent magnons leads to soliton formation. Having in mind that the scattering of CEs with phonons in the aforementioned temperature range is negligible, we expect that measurments of electrical resistivity of MS as a function of AEMF will reveal the features predicated by law presented by formula (17).

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