POLARISATION FLUCTUATIONS IN NONLINEAR OPTICAL FIBRES

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1. INTRODUCTION

Nonlinear optical fibres are systems in which nonlinearity and dispersion give rise to the existence of stable solitary envelope pulses associated with light at a given carrier frequency [1]. The direct observability of the formation and propagation of these solitary pulses and the prospect of their application in long-distance telecommunication systems has stimulated a large number of theoretical investigations in this field [2]. Mathematically, pulse propagation in the monomode regime of an axisymmetric optical fibre can be described by a pair of nonlinearly coupled nonlinear Schrödinger equations (NLS). The two complex variables in these equations play the role of slowly varying amplitudes of two degenerate fibre modes (i.e. fibre modes having the same propagation constant k for given frequency ω). Unitary transformations of the complex two-component vector of these amplitudes leave the intensity unchanged but affect phase and polarisation of a pulse. If the coupled NLS equations are invariant under U(2)-transformations, then the system is integrable via the inverse scattering transform [3]. It possesses a multi-parameter family of pulse solutions which have been termed vector solitons [4]. For real material coefficients, the nonlinear terms in the coupled equations are not invariant under U(2)-transformations and the system is not integrable [5]. It does however possess a multi-parameter family of solitary wave solutions [6] which may be regarded as generalisations of vector solitons and are, for special polarisations, identical to them. It will be shown in detail that material inhomogeneities lead to extra terms in the evolution equations that generate unitary transformations of the two-component vector of complex amplitudes in the linear limit. If the system of coupled NLS equations is integrable, the effect of these terms is trivial, while in the general case, they give rise to interesting behaviour of pulse propagation in which the aforementioned solitary pulse solutions play an important role.

2. VARIATIONS OF THE LINEAR REFRACTIVE INDEX

To derive evolution equations for the propagation of envelope pulses in nonlinear optical fibres, we start from the wave equation

$$\nabla\times(\nabla\times\underset{\sim}{E}) + \frac{\partial^2}{\partial t^2}(\varepsilon+N|\underset{\sim}{E}|^2)\underset{\sim}{E} = 0 \tag{2.1}$$

for the electric field $\underset{\sim}{E}$, which we expand in powers of a small parameter ν,

$$\underset{\sim}{E}(\underset{\sim}{x},t) = e^{i(kz-\omega t)}\sum_{n=1}^{\infty} \nu^n \underset{\sim}{E}_n(\underset{\sim}{x},t) + c.c. \tag{2.2}$$

We also introduce stretched coordinates $Z_n=\nu^n z$, $T_n=\nu^n t$, where z is the coordinate in the fibre direction. In axisymmetric fibres which are homogeneous along the z-direction, the linear dielectric constant ε and the Kerr coefficient N depend on the radial coordinate r only and the linearised system allows $\underset{\sim}{E}$ to be a linear combination of the modal fields $\underset{\sim}{E}_\pm(r,\theta)$ of two degenerate fibre modes which we choose to be of circular polarisation, so that

$$\underset{\sim}{E}_1(\underset{\sim}{x},t) = \sum_{\sigma=\pm} \underset{\sim}{E}_\sigma(r,\theta)\, A_\sigma(Z_1,T_1,Z_2,..) . \tag{2.3}$$

In the following, we also take into account small and gradual variations of ε that may violate the axial symmetry, by decomposing $\varepsilon(\underset{\sim}{x})=\varepsilon_1(r)+\varepsilon_2(r,\theta,z)$ and scaling $\varepsilon_2=\nu^m\delta(r,\theta,Z_2)$. For m=2, the variations of the dielectric constant due to inhomogeneities are of the same order in ν as its variations induced by the Kerr nonlinearity. Following the standard procedure of multiple scales, simplified by the use of compatibility conditions for the equations of third order in ν [7], evolution equations are obtained for the amplitudes $A_\pm$ which, after rescaling, take the form

$$i\frac{\partial}{\partial z}A_\pm = \frac{\partial^2}{\partial t^2}A_\pm + \sum_{\sigma=\pm} \Lambda_{\pm\sigma}(z)A_\sigma + (|A_\pm|^2+ h|A_\mp|^2)A_\pm \quad , \tag{2.4}$$

where we now use the symbols z and t for a length proportional to Z_2 and a retarded time variable proportional to T_1-Z_1/S, S being the group velocity of the linear degenerate fibre modes with $\varepsilon_2 \equiv 0$. For h=1, the system (2.4) is integrable [3]. However, for weakly guiding fibres, the material coefficient h takes values close to 2, and the system is <u>not</u> integrable [5]. The components of the Hermitian matrix $\underset{\sim}{\Lambda}$ are proportional to overlap integrals of the function $\delta(r,\theta,Z_2)$ with products of two modal fields, of the form

$$\Lambda_{++}=\Lambda_{--}\sim \int_0^\infty f_0(r) \int_0^{2\pi} \delta(r,\theta,Z_2)\; d\theta\; dr \tag{2.5}$$

$$\Lambda_{+-}=\Lambda^*_{-+}\sim \int_0^\infty f_1(r) \int_0^{2\pi} e^{-2i\theta}\delta(r,\theta,Z_2)\; d\theta\; dr\ . \tag{2.6}$$

Due to the symmetry of the problem, the diagonal elements are equal. They can be different if δ is allowed to be a tensor. In the absence of nonlinearity, the matrix $\underset{\sim}{\Lambda}$ causes the two-component vector (A_+,A_-) to undergo continuous unitary transformations. In the integrable case h=1, this matrix may be eliminated from the evolution equations by transformation to the variables $B_\pm(z,t)=\sum_{\sigma=\pm} R_{\sigma\pm}(z)A_\sigma(z,t)$ with the unitary matrix $\underset{\sim}{R}$ satisfying the ordinary differential equation

$$-i\frac{d}{dz}\underset{\sim}{R} = \underset{\sim}{\Lambda}^*\underset{\sim}{R}\quad . \tag{2.7}$$

If h≠1, this is no longer possible. The diagonal elements of $\underset{\sim}{\Lambda}$ (if they are equal) may however still be absorbed by a redefinition of phase, so that we may assume without loss of generality that $\Lambda_{++}=\Lambda_{--}=0$.

3. SCATTERING OF A PULSE AT A BIREFRINGENCE DEFECT

We now investigate the effect on pulse propagation of an irregularity in ε causing Λ_{+-} to be significantly non-zero only in a finite interval. We shall call such a localised irregularity a birefringence defect. The simple functional form $\Lambda_{+-}(z)=iS\exp\{-(z-z_0)^2/w^2\}$ with $z_0 \gg w$ is chosen, and for z=0, we assume a pulse of the form $A_+(0,t)=a(0,t)=\sqrt{2}\,\mathrm{sech}(t)$, $A_-(0,t)=0$, where a(z,t) is a soliton solution of the single NLS equation describing a circularly polarised mode. In the integrable case the solution takes exactly the form $(A_+(z,t),\ A_-(z,t))= (\cos\alpha,\ \sin\alpha)\ a(z,t)$, where $\alpha=\alpha(z)$ has the limit $-\sqrt{\pi}Sw$ for $z \gg z_0+w$. The effect of a birefringence defect thus consists in transforming a vector soliton into another one with altered polarisation which depends only on the product Sw.

Although the system is not integrable for $h\neq 1$, it possesses solitary wave solutions corresponding to simple pulses [6],

$$A_{\pm}(z,t) = \eta F_{\pm}(\eta t;\alpha)\ \exp(-i\eta^{2}\beta_{\pm}z)\ . \qquad (3.1)$$

The real functions $F_{\pm}(\tau;\alpha)$ are solutions of the ordinary differential equations

$$\frac{d^2}{d\tau^2}F_{\pm} = (\beta_{\pm}-F_{\pm}^{2}-hF_{\mp}^{2})F_{\pm} \qquad (3.2)$$

decaying to zero exponentially as $\tau\to\pm\infty$. The parameter α plays the role of a polarisation angle if we define $\tan\alpha=F_{-}(0)/F_{+}(0)$ and $F_{\pm}$ are even functions. Numerical integrations have been carried out with the above "initial conditions" at $z=0$. Their results suggest that, after passing the birefringence defect, the pulse evolves into a solution of the coupled NLS equations being predominantly of the form (3.1), (3.2). This has been tested by using a consistency relation between $\beta_{\pm}$ and $F_{\pm}(0)$ following from the nonlinear eigenvalue problem (3.2). In addition, superimposed oscillations and continuous output of radiation has been observed for sufficiently large S. The numerical integrations have been performed for different combinations of S and w, but with fixed product $Sw=0.2$. For $h=1$, the limiting behaviour of $A_{\pm}$ for $z\gg z_0+w$ is then identical for all these combinations.

Different behaviour is found for $h=2$. Here, two regimes may be distinguished. For large S and small width w on the length scale on which nonlinearity and dispersion are effective, the main effect of the birefringence irregularity is to reset the initial conditions for the evolution of $A_{\pm}$. This is illustrated in Fig. 1, which shows the behaviour of the maxima of $|A_{\pm}(t)|$ as functions of z. This resetting of initial conditions, and also the subsequent evolution, will only depend on the product Sw. (It should be noted that exchange of intensity between the two modes can take place only if Λ_{+-} is nonzero.) With increasing w and decreasing S, less intensity is converted from the first (+) to the second (−) mode and finally, A_{-} is nonzero only in the neighbourhood of z_0. In other words, the pulse regains its initial polarisation after having passed the defect, in strong contrast to the integrable case. This qualitatively different behaviour can be understood in the framework of soliton perturbation theory (recently applied to similar problems in optical fibres in refs. [8-11]) based on the assumption that, if w is large on the scale on

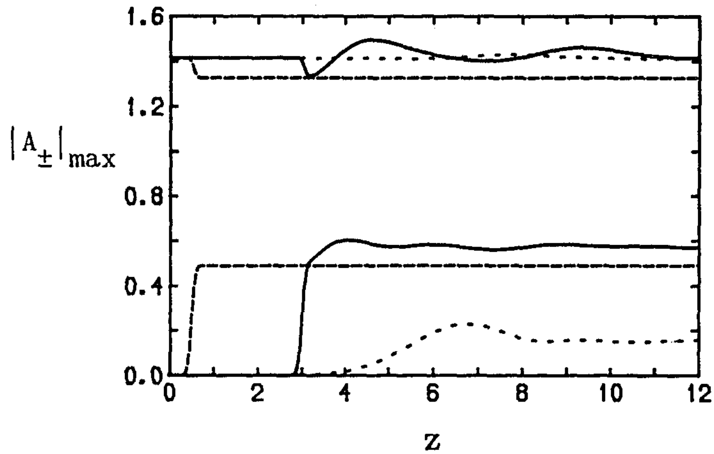

Fig. 1. Maxima of $|A_\pm(t)|$ as functions of z. In all three cases Sw=0.2. dashed: h=1, w=0.1; solid: h=2, w=0.1; dotted: h=2, w=1.6.

which nonlinear evolution takes place and S is small, the fields $A_\pm$ can be approximated by the functional form

$$A_\pm(z,t) = \eta(z)\ F_\pm(\eta(z)t;\alpha(z))\ \exp\{-i[\phi(z)\pm\Psi(z)/2]\} \tag{3.3}$$

with parameters η, α, ϕ and Ψ varying slowly along the fibre. By inserting this Ansatz into the action integral for $A_\pm$ and taking the variation with respect to these parameters, coupled equations of the form

$$\frac{d}{dz}\alpha = \lambda\ K_1(\alpha)\ \sin(\Psi+\gamma)\ , \qquad \frac{d}{dz}\Psi = K_2(\alpha) + \lambda\ K_3(\alpha)\ \cos(\Psi+\gamma) \tag{3.4}$$

are obtained, where λ and γ are the modulus and argument of Λ_{+-}. In the integrable case, the term K_2 in (3.4) is absent. For a qualitative discussion, we confine ourselves to the case $|h-1|<<1$. Then, η is approximately a constant and $K_2(\alpha)\approx(4/3)(1-h)\eta^2\cos(2\alpha)$. After a transformation to linearly polarised modes, coupled equations are obtained of the form derived earlier for $\gamma=0$ and λ constant [8,10]. These may be linearised around the initial pulse parameters to yield driven harmonic oscillator equations

$$\frac{d^2}{dz^2}q_\pm + \Omega^2 q_\pm = f_\pm \tag{3.5}$$

for variables $q_\pm$ connected with the polarisation angle α via $\alpha^2 \approx q_+^2 + q_-^2$. The driving forces $f_\pm$ are linear combinations of the real and imaginary parts of Λ_{+-} and their derivatives with respect to z, and the "frequency" is

$\Omega=(4/3)|h-1|\eta^2$. The deviation of h from 1 thus gives rise to a restoring force that causes the polarisation angle to return to its initial value 0 after the pulse has passed the birefringence defect. This behaviour does not seem to occur for initially linearly polarised pulses with $A_+=A_-$.

With the initial conditions $a(0,t)=2\sqrt{2}\text{sech}(t)$ a two-soliton bound state of the NLS equation evolves. In distinction to the single soliton case, the effect of a birefringence defect now depends on its location z_0 relative to the stage of periodic internal oscillation of the two-soliton bound state. Results of a numerical integration for h=2 and a defect with width w=0.1 and strength S=2 at $z_0=0.78$ indicate that as the pulse encounters the defect, it strongly distorts producing a large amount of radiation. In a second phase, an extensive reshaping of the pulse takes place with lesser generation of radiation. The internal oscillations gradually disappear and a sharp central peak emerges with two broad wings detaching from it (Fig.2). Analysis of

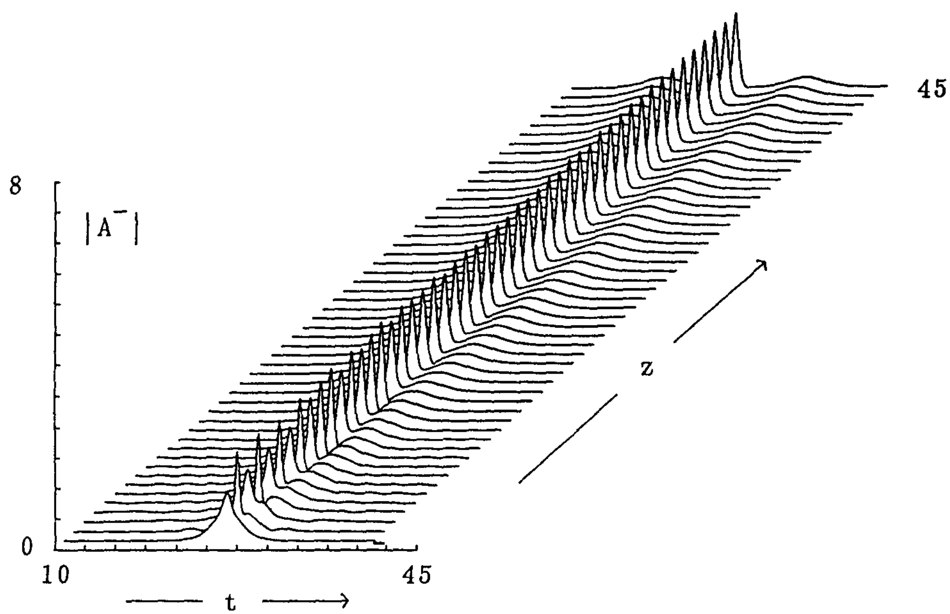

Fig. 2. Evolution of $|A_-|$ of a pulse initially corresponding to a two-soliton bound state scattered at a birefringence defect at position $z_0=0.78$ marked by a little bar.

the moduli and arguments of $A_\pm$ at maximum suggests that the central peak corresponds to a solitary wave solution of the type (3.1) with (3.2). These findings compare with earlier numerical results by Blow et al. [12] for a fibre with constant birefringence, who found that a two-soliton bound state evolves into a highly compressed single pulse.

4. STRONG BIREFRINGENCE

The evolution equations (2.4) have been obtained with the assumption that the inhomogeneous part ε_2 of the permittivity is of second order in ν (m=2). We now consider briefly the case m=1. In order to keep the multiple scales expansion uniform [13], we introduce coordinates $\zeta_j=f_j(Z_2)/\nu$ with $f_1'(Z_2)=\lambda(Z_2)$ and $f_2'(Z_2)=\Lambda_{++}(Z_2)$. We then perform the transformation

$$A_\pm(\zeta_1,\zeta_2,Z_2,..) = \sum_{\sigma=\pm} R_{\pm\sigma}(\zeta_1,\zeta_2)\ B_\sigma(Z_2,..) \ , \tag{4.1}$$

where the unitary matrix $\underset{\sim}{R}$ has elements

$$R_{\sigma\sigma'} = \exp\{i(\Psi_{\sigma\sigma'}-\zeta_2)\}/\sqrt{2} \ , \quad \Psi_{++}= \gamma-\zeta_1 =-\Psi_{--}+\pi \ , \quad \Psi_{-+}=-\zeta_1 =-\Psi_{+-} \ . \tag{4.2}$$

Following now the procedure of multiple scales, a pair of evolution equations emerges for $B_\pm$, from which all fast oscillations on the scales of $\zeta_{1,2}$ are eliminated. After rescaling in the same way as in the derivation of (2.4), they take the form

$$i\frac{\partial}{\partial z}B_\pm = \Pi_\pm(z)B_\pm+iM_\pm(z)\frac{\partial}{\partial t}B_\pm+\frac{\partial^2}{\partial t^2}B_\pm+\frac{1}{2}(1+h)\{|B_\pm|^2+\frac{2}{1+h}|B_\mp|^2\}B_\pm \ , \tag{4.3}$$

where $\Pi_\pm$ and $M_\pm$ are real functions. The quantities $M_\pm$ may be expressed in terms of the $\Lambda_{\sigma\sigma'}$ and their first derivatives with respect to the propagation constant k (prior to rescaling). By simple transformations, the first term on the right-hand side of (4.3) can be eliminated and the group velocity shifts $M_\pm(z)$ can be arranged to have equal modulus and opposite signs. For the case of homogeneous fibres, equations of this type have been obtained by Menyuk [14,15] and pulse propagation in this case has been studied numerically [15,16] and analytically [11] on the basis of such equations. For constant $M_\pm$ the second term on the right-hand side of (4.3) can be eliminated by a simple phase transformation, and the resulting pair of coupled NLS equations has solitary pulse solutions of the form (3.1) with altered coefficients in the nonlinear terms of (3.2).

In the preceding discussions, we have considered symmetry-breaking of the refractive index which varies gradually (on the scale of Z_2) along the fibre. If the refractive index fluctuates on length scales considerably shorter than the characteristic length ℓ for the pulse evolution due to nonlinearity and dispersion, the effect of such fluctuations is primarily to introduce effective coefficients in the evolution equations for slowly varying amplitudes as averages of the fluctuations over lengths smaller than ℓ. In an approximation, these averages may then be calculated as averages over an ensemble of different realisations [17]. Gaussian statistical properties of the fluctuating part ϵ_2 of the dielectric constant translate into a Gaussian ensemble for the matrix $\underset{\sim}{\Lambda}$. For systems in which ϵ_2 is of first order in ν and preserves the axial symmetry <u>on average</u>, evolution equations have been derived for slowly varying amplitudes $B_\pm$ which are related to $A_\pm$ by a unitary transformation of the form (4.1) [18]. These evolution equations are of the same form as those in the absence of fluctuations. However, the coefficients in front of the nonlinear terms acquire corrections determined by the correlation function $\langle \Lambda_{+-}(z)\Lambda_{-+}(z')\rangle$.

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