Dissipation in Quantum Field Theory

E. Celeghini[(1)], M. Rasetti[(2)] and G. Vitiello[(3)]

(1) Dipartimento di Fisica dell'Universitá and I.N.F.N., Firenze , Italy

(2) Dipartimento di Fisica and I.N.F.M., Politecnico di Torino, Italy

(3) Dipartimento di Fisica, Universitá di Salerno and I.N.F.N., Sezione di Napoli, Italy

A major difficulty that appears in the study of dissipative systems in Quantum Mechanics is that the canonical commutation relations (*CCR*) are not preserved by time evolution due just to damping terms. Then one introduces fluctuating forces in order to preserve the quantum mechanical consistency, namely the canonical structure. Another way to handle the problem is to start from the beginning with an Hamiltonian which describes the system, the bath and the system-bath interaction. Subsequently, one eliminates the bath variables which originate both damping and fluctuations, thus obtaining the reduced density matrix.

Purpose of the present paper is to discuss some aspects of dissipation in Quantum Field Theory (*QFT*) resorting just to the relatively simple example of the damped harmonic oscillator, whose classical equation of motion is

$$m\ddot{x} + \gamma\dot{x} + \kappa x = 0 \quad . \tag{1}$$

Contrary to the more traditional attitude by which one tries to accomodate the non-unitary character of time-evolution implied by dissipation in the familiar framework of the unitary operator algebra, our task here is to provide a picture of non-unitary evolution at a quantum level without forcing or reducing it to the framework of unitary operator algebra. In the following we show that in *QFT* there is enough room to produce such a picture if one takes advantage of the existence of infinitely many unitarily inequivalent representations of the *CCR*. As a prize for such an unconvenctional attitude, the statistical nature of dissipative phenomena naturally emerges from our formalism and some light is shed on the question of if and where the arrow of time can be find at microscopic level.

In order to deal with an isolated system, as the canonical quantization scheme requires, a procedure of doubling of the phase-space dimension is necessary[1]. Thus the lagrangian for system (1) is written as

$$L = m\dot{x}\dot{y} + \frac{1}{2}\gamma(x\dot{y} - \dot{x}y) - \kappa xy \quad , \tag{2}$$

where y denotes the position variable for the *doubled* system. Intuitively one expects the y variable to grow as rapidly as the x solution decays: in this sense y may be thought of as describing an effective degree of freedom for the heat bath to which the system (1) is coupled. (1) is obtained by varying (2) with respect to y, whereas variation with respect to x gives indeed $m\ddot{y} - \gamma\dot{y} + \kappa y = 0$, which appears in fact to be the *time reversed* ($\gamma \rightarrow -\gamma$) of (1). The canonical momenta p_x and p_y (the collection of dynamical variables $\{x, p_x, y, p_y\}$ spans the new phase-space) are then given by $p_x \equiv \frac{\partial L}{\partial \dot{x}} = m\dot{y} - \frac{1}{2}\gamma y$; $p_y \equiv \frac{\partial L}{\partial \dot{y}} = m\dot{x} - \frac{1}{2}\gamma x$. Canonical quantization may then be performed by introducing the commutators $[x, p_x] = i\hbar = [y, p_y]$, $[x, y] = 0 = [p_x, p_y]$, and the corresponding sets of annihilation and creation operators $a, a^\dagger, b$ and $b^\dagger$. Performing the linear canonical transformation $A \equiv \frac{1}{\sqrt{2}}(a + b)$, $B \equiv \frac{1}{\sqrt{2}}(a - b)$, the quantum hamiltonian is obtained

$$\begin{aligned} \mathcal{H} &= \mathcal{H}_0 + \mathcal{H}_I \quad , \\ \mathcal{H}_0 = \hbar\Omega(A^\dagger A - B^\dagger B) \quad &, \quad \mathcal{H}_I = i\hbar\Gamma(A^\dagger B^\dagger - AB) \quad , \end{aligned} \tag{3}$$

where $\Gamma \equiv \frac{\gamma}{2m}$ is the decay constant for the classical variable $x(t)$. It is easy to realize that the dynamical group structure associated with our system of coupled quantum oscillators is that of $SU(1,1)$. The two mode realization of the algebra $su(1,1)$ is indeed generated by

$$J_+ = A^\dagger B^\dagger \quad , \quad J_- = J_+^\dagger = AB \quad , \quad J_3 = \frac{1}{2}(A^\dagger A + B^\dagger B + 1) \quad , \tag{4}$$

corresponding to the Casimir operator $\mathcal{C}$ defined as: $\mathcal{C}^2 \equiv \frac{1}{4} + J_3^2 - \frac{1}{2}(J_+J- + J_-J_+)$ $= \frac{1}{4}(A^\dagger A - B^\dagger B)^2$. Note that $[\mathcal{H}_0, \mathcal{H}_I] = 0$, as $\mathcal{H}_0$ is in the center of the dynamical algebra.

Let us denote by $\{|n_A, n_B>\}$ the set of simultaneous eigenvectors of $A^\dagger A$ and $B^\dagger B$, with n_A, n_B non-negative integers. Let the initial state be the *vacuum* $|n_A = 0, n_B = 0> \equiv |0>$, such that $A|0>= 0 = B|0>$, its time-evolution is given by

$$\begin{aligned} |0(t)> &= \exp\left(-it\frac{\mathcal{H}}{\hbar}\right)|0>= \exp\left(-it\frac{\mathcal{H}_I}{\hbar}\right)|0> \\ &= \frac{1}{\cosh(\Gamma t)} \exp\left(\tanh(\Gamma t) J_+\right)|0> \quad , \end{aligned} \tag{5}$$

namely a two-mode Glauber coherent state [1, 2, 3] (*i.e.* a generalized coherent state for $su(1,1)$). Eq. (5) shows that at every time t the state $|0(t)>$ has unit norm, however as $t \to \infty$ it gives rise to an asymptotic state which is orthogonal to the initial state $|0> : <0(t)|0(t)>= 1$ and

$$\lim_{t\to\infty} <0(t)|0>= \lim_{t\to\infty} \exp\left(-\ln\cosh(\Gamma t)\right) \to 0 \quad . \tag{6}$$

Eq. (6) expresses the instability (*decay*) of the vacuum under time evolution operator $\mathcal{U} \equiv \exp\left(-it\frac{\mathcal{H}_I}{\hbar}\right)$. We reach thus the conclusion that as an effect of damping (recall that $\mathcal{H}_I \to 0$ as $\gamma \to 0$) the time-evolution generator $\mathcal{H}_I$ leads outside the original Hilbert space of states. As the ordinary quantization procedure requires that a definite representation of the *CCR* should be used in order to describe the system under study, we say that the time-evolution generator of the damped oscillator, by leading outside the original Hilbert space, produces *unquantization.* This is an obstruction which is to be bypassed in producing a canonical scheme. One should notice that so far the problem was tackled within the framework of Quantum Mechanics, namely within a scheme were the von Neumann theorem only allows unitarily equivalent representations of *CCR*. In what follows, we intend to show how the pathology exhibited by (6) can be controlled if one operates in a different scheme, giving up in particular the condition of finiteness of the number of degrees of freedom. This, in turn, is obviously equivalent to moving to a second quantization scheme, *i.e.* to a *QFT*, where the infinite number of degrees of freedom allows the coexistence of infinitely many unitarily inequivalent representations of the *CCR*'s. The most straightforward extension to *QFT* of the hamiltonian given by eq. (3), describing an (infinite) collection of damped harmonic oscillators, is

$$\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_I \quad ,$$
$$\mathcal{H}_0 = \sum_\kappa \hbar\Omega_\kappa \left(A_\kappa^\dagger A_\kappa - B_\kappa^\dagger B_\kappa\right) \quad , \quad \mathcal{H}_I = i \sum_\kappa \hbar\Gamma_\kappa \left(A_\kappa^\dagger B_\kappa^\dagger - A_\kappa B_\kappa\right) \quad , \tag{7}$$

where κ labels the field degrees of freedom, *e.g.* spatial momentum. As usual, the computational strategy is now to work at finite volume of the system V, and to perform at the end the limit $V \to \infty$. The commutation relations are:

$$[\, A_\kappa, A_\lambda^\dagger \,] = \delta_{\kappa,\lambda} = [\, B_\kappa, B_\lambda^\dagger \,] \quad ; \quad [\, A_\kappa, B_\lambda^\dagger \,] = 0 = [\, A_\kappa, B_\lambda \,] \quad . \tag{8}$$

We still have $[\, \mathcal{H}_0, \mathcal{H}_I \,] = 0$, and corresponding to eq. (5) we have (formally, at finite volume V),

$$|0(t)> = \prod_\kappa \frac{1}{\cosh(\Gamma_\kappa t)} \exp\left(\tanh(\Gamma_\kappa t) J_+^{(\kappa)}\right)|0> \quad , \tag{9}$$

with $J_+^{(\kappa)} \equiv A_\kappa^\dagger B_\kappa^\dagger$. Moreover $<0(t)|0(t)> = 1 \;\; \forall t$, and

$$<0(t)|0> = \exp\left(-\sum_\kappa \ln\cosh(\Gamma_\kappa t)\right) \quad ; \tag{10}$$

which shows how, provided $\sum_\kappa \Gamma_\kappa > 0$,

$$\lim_{t\to\infty} <0(t)|0> \propto \lim_{t\to\infty} \exp\left(-t\sum_\kappa \Gamma_\kappa\right) = 0 \quad . \tag{11}$$

Now, using the customary continuous limit relation $\sum_\kappa \mapsto \frac{V}{(2\pi)^3}\int d^3\kappa$, in the infinite-volume limit we have (for $\int d^3\kappa\, \Gamma_\kappa$ finite and positive)

$$\begin{aligned} &<0(t)|0> \underset{V\to\infty}{\longrightarrow} 0 \quad \forall t \quad , \\ &<0(t)|0(t')> \underset{V\to\infty}{\longrightarrow} 0 \quad \forall t, t' \quad , \quad t \neq t' \quad . \end{aligned} \tag{12}$$

We notice that the time-evolution transformations

$$\begin{aligned} A_\kappa \mapsto A_\kappa(t) &= \mathrm{e}^{-i\frac{t}{\hbar}\mathcal{H}_I} A_\kappa \mathrm{e}^{i\frac{t}{\hbar}\mathcal{H}_I} = A_\kappa \cosh(\Gamma_\kappa t) - B_\kappa^\dagger \sinh(\Gamma_\kappa t) \quad ; \\ B_\kappa \mapsto B_\kappa(t) &= \mathrm{e}^{-i\frac{t}{\hbar}\mathcal{H}_I} B_\kappa \mathrm{e}^{i\frac{t}{\hbar}\mathcal{H}_I} = -A_\kappa^\dagger \sinh(\Gamma_\kappa t) + B_\kappa \cosh(\Gamma_\kappa t) \quad , \end{aligned} \tag{13}$$

and their hermitian conjugates can each be implemented, for every κ, as inner automorphism for the algebra $su(1,1)_\kappa$. Such an automorphism is nothing but the well known Bogolubov transformations. The transformations (13) are canonical, as they preserve the *CCR* (8). So, for each t we have a copy $\{A_\kappa(t), A_\kappa^\dagger(t), B_\kappa(t), B_\kappa^\dagger(t);\ |0(t)> \ |\forall\kappa\}$ of the original algebra and of its highest weight vector $\{A_\kappa, A_\kappa^\dagger, B_\kappa, B_\kappa^\dagger;\ |0> \ |\forall\kappa\}$, induced by the time evolution operator. The time evolution operator can therefore be thought of as a generator of the group of automorphisms of $\bigoplus_\kappa su(1,1)_\kappa$ parametrized by time t.

It is very important to point out that the various copies need not to be unitarily equivalent representations of the *CCR*'s; as a matter of fact, they do become unitarily inequivalent in the infinite-volume limit, as shown in (12). This implies that the automorphisms (13) are defined up to arbitrary intertwining operators; in fact, one should more accurately say that the dynamical algebra $\mathcal{X}$ is given globally by the doubly-continuous direct sum $\bigoplus_t \bigoplus_\kappa \mathcal{A}^{\kappa(t)} \circ su(1,1)_\kappa$, where $\mathcal{A}^{\kappa(t)}$ denotes the intertwining operator [4, 5, 8] connecting the representations realizable at time t.

As a direct check, one can easily verify how at each time t one has

$$A_\kappa(t)|0(t)> = 0 = B_\kappa(t)|0(t)> \quad , \quad \forall t \quad .$$

The number of modes of type A_κ is given, at each instant t by

$$\mathcal{N}_{A_\kappa} \equiv <0(t)|A_\kappa^\dagger A_\kappa|0(t)> = \sinh^2(\Gamma_\kappa t) \quad ; \tag{14}$$

and similarly for the modes of type B_κ.

The state $|0(t)>$ on the other hand may be written as:

$$\begin{aligned} |0(t)> &= \exp\left(-\frac{1}{2}\mathcal{S}_A\right) \exp\left(\sum_\kappa A_\kappa^\dagger B_\kappa^\dagger\right)|0> \\ &= \exp\left(-\frac{1}{2}\mathcal{S}_B\right) \exp\left(\sum_\kappa B_\kappa^\dagger A_\kappa^\dagger\right)|0> \quad , \end{aligned} \tag{15}$$

with

$$\mathcal{S}_A \equiv -\sum_\kappa \left\{ A_\kappa^\dagger A_\kappa \ln\sinh^2(\Gamma_\kappa t) - A_\kappa A_\kappa^\dagger \ln\cosh^2(\Gamma_\kappa t) \right\} \quad ; \tag{16}$$

and $\mathcal{S}_B$ given by the same expression with B_κ and $B_\kappa^\dagger$ replacing A_κ and $A_\kappa^\dagger$, respectively. As A_κ's and B_κ's commute, due to (15) we shall simply write $\mathcal{S}$ for either $\mathcal{S}_A$ or $\mathcal{S}_B$.

From (15) one derives the expansion $|0(t)>=\sum_{n\geq 0}\sqrt{W_n(t)}\,|n,n>$, where n denotes the multi-index $\{n_\kappa\}$, with $\sum_{n\geq 0} W_n(t)=1$,

$$<0(t)|\mathcal{S}|0(t)>=-\sum_{n\geq 0} W_n(t)\ln W_n(t) \quad . \tag{17}$$

Eq. (17) leads us therefore to interpreting $\mathcal{S}$ as the *entropy* for the dissipative system. On the other hand we have, for the time variation of $|0(t)>$

$$\frac{\partial}{\partial t}|0(t)>=-\frac{i}{\hbar}\mathcal{H}_I|0(t)> \quad . \tag{18}$$

Use of eqs. (7) and (16) shows then that

$$\frac{\partial}{\partial t}|0(t)>=-\left(\frac{1}{2}\frac{\partial\mathcal{S}}{\partial t}\right)|0(t)> \quad , \tag{19}$$

which may also be derived directly from (15). Equation (19) shows that $i\left(\frac{1}{2}\hbar\frac{\partial\mathcal{S}}{\partial t}\right)$ is the generator of time-translations, namely time-evolution is controlled by the entropy variations. It appears to us suggestive that for a dissipative quantum system the same operator that controls time evolution could be interpreted as defining a dynamical variable whose expectation value is formally an entropy: we conjecture that the connection between these features of $\mathcal{S}$ reflects correctly the irreversibility of time evolution characteristic of dissipative motion[6]. Damping (or, more generally, dissipation) implies indeed the choice of a privileged direction in time evolution (*time arrow*) with a consequent breaking of time-reversal invariance.

We also observe that $<0(t)|\mathcal{S}|0(t)>$ grows monotonically with t from value 0 at $t=0$ to infinity at $t=\infty$; *i.e.* the entropy for both A and B increases as the system evolves in time towards the stability condition at $t=\infty$. Moreover the difference $\mathcal{S}_A-\mathcal{S}_B$ of the A- and B-entropies is constant in time: $[\mathcal{S}_A-\mathcal{S}_B,\mathcal{H}]=0$. Since the B-particles are the holes for the A-particles, $\mathcal{S}_A-\mathcal{S}_B$ turns out to be, in fact, the (conserved) entropy for the complete system.

In conclusion, the system in its evolution runs over a variety of representations of the CCR's which are unitarily inequivalent to eac h other for t $\neq$ t' in the infinite-volume limit. It is in fact the non-unitary character of time-evolution implied by damping which is recovered, in a consistent scheme, in the unitary inequivalence among representations at different times in the infinite-volume limit.

Also, the statistical nature of dissipative phenomena naturally emerges from our formalism, even though no statistical concepts were introduced a priori: for example, the *entropy operator* enter s the picture as time evolution generator. It is therefore an interesting question asking ourselves whether and how such statistical features may actually be related to thermal concepts. Let us clarify this point in the following discussion.

Let us focus, for the sake of definiteness, on the A-modes, and introduce the functional

$$\mathcal{F}_A\equiv<0(t)|\left(\mathcal{H}_A-\frac{1}{\beta}\mathcal{S}_A\right)|0(t)> \quad . \tag{20}$$

β is a strictly positive function of time to be determined; $\mathcal{H}_A$ is the part of $\mathcal{H}_0$ relative to the A-modes only, namely $\mathcal{H}_A = \sum_\kappa \hbar\Omega_\kappa A_\kappa^\dagger A_\kappa$. We write $\vartheta_\kappa \equiv \Gamma_\kappa t$, and look for the values of ϑ_κ rendering $\mathcal{F}_A$ stationary:

$$\frac{\partial \mathcal{F}_A}{\partial \vartheta_\kappa} = 0 \quad ; \quad \forall \kappa \quad . \tag{21}$$

Condition (21) is clearly a stability condition to be satisfied for each representation. Setting $E_\kappa \equiv \hbar\Omega_\kappa$, it gives

$$\beta(t) E_\kappa = -\ln \tanh^2(\vartheta_\kappa) \quad . \tag{22}$$

From (22) we have then

$$\mathcal{N}_{A_\kappa}(t) = \sinh^2(\Gamma_\kappa t) = \frac{1}{e^{\beta(t)E_\kappa} - 1} \quad ; \tag{23}$$

which is the Bose distribution for A_κ at time t provided we assume $\beta(t)$ to represent the inverse temperature $\beta(t) = \frac{1}{k_B T(t)}$ at time t (k_B denotes the Boltzmann constant). This allows us to recognize $\{|0(t)>\}$ as a representation of the *CCR*'s at finite temperature, equivalent – up to an arbitrary choice of the temperature scale, which is a differentiable function of time – with the Thermo-Field Dynamics representation $\{|0(\beta)>\}$ of Umezawa and Takahashi[7].

We can now therefore interpret $\mathcal{F}_A$ as the free energy and $\mathcal{N}_A$ as the average number of activated A-modes at the temperature defined by time t through the function $\beta(t)$. In function of the time, the change in the energy $E_A \equiv \sum_\kappa E_\kappa \mathcal{N}_{A_\kappa}$ is given by the relation $dE_A = \sum_\kappa E_\kappa \dot{\mathcal{N}}_{A_\kappa} dt$; as one should expect, since the time evolution induces transitions over different representations, which in turn imply changes in the number of activated modes. Time derivative of $\mathcal{N}_{A_\kappa}$ is of course obtained from eq. (14). We can also compute the change in entropy [8]

$$d\mathcal{S}_A = \frac{\partial}{\partial t}\Big(< 0(t)|\mathcal{S}_A|0(t) >\Big) \quad = \quad \beta \sum_\kappa \hbar\Omega_\kappa \dot{\mathcal{N}}_{A_\kappa}(t) dt \quad = \quad \beta dE_A(t) \quad . \tag{24}$$

Eq. (24) shows that

$$dE_A - \frac{1}{\beta} d\mathcal{S}_A = 0 \quad . \tag{25}$$

When changes in inverse temperature are slow namely $\frac{\partial \beta}{\partial t} = -\frac{1}{k_B T^2}\frac{\partial T}{\partial t} \approx 0$ (which is the case for adiabatic variations of temperature, at T high enough), eq. (25) can be obtained directly by minimizing the free energy (20) $d\mathcal{F}_A = dE_A - \frac{1}{\beta} d\mathcal{S}_A = 0$; which – by reference to conventional thermodynamics – allows us to recognize E_A as the internal energy of the system. It also expresses the first principle of thermodynamics for a system coupled with environment at constant temperature and in absence of mechanical work.

We may also define as usual heat as $dQ = \frac{1}{\beta} dS$. We thus see that the change in time $d\mathcal{N}_A$ of particles condensed in the vacuum turns out into heat dissipation dQ.

Let us close this paper with few remarks[8]. The total Hamiltonian (7) is invariant under the transformations generated by $J_2 = \bigoplus_\kappa J_2^{(\kappa)}$. The vacuum however is not invariant under J_2 (see eq.(9)) in the infinite volume limit. Moreover, at each time t, the representation $|0(t)>$ may be characterized by the expectation value in the state $|0(t)>$ of, *e.g.*, $J_3^{(\kappa)} - \frac{1}{2}$: thus the total number of particles $n_A + n_B = 2n$ can be taken as an order parameter. Therefore, at each time t the symmetry under J_2 transformations is spontaneously broken. On the other hand, $\mathcal{H}_I$ is proportional to J_2. Thus, in addition to breakdown of time-reversal (discrete) symmetry, already mentioned above, we also have spontaneous breakdown of time translation (continuous) symmetry.

In other words we led dissipation (i.e. energy non-conservation), as it has been described in this paper, to an effect of breakdown of time translation and time-reversal symmetry. It is an interesting question asking which is the mode playing the role of the Goldstone mode related with the breakdown of continuous time translation symmetry: we observe that since $n_A - n_B$ is constant in time, the condensation (annihilation and/or creation) of AB-pairs does not contributes to the vacuum energy so that AB-pair may play the role of Goldstone mode.

From the point of view of boson condensation, time evolution in the presence of damping may be then thought of as a sort of continuous transition among different phases, each phase corresponding, at time t, to the representation $|0(t)>$ and characterized by the value of the order parameter at the same time t. The damped oscillator thus provides an archetype of system undergoing continuous phase transition.

We also observe that in perturbation theory a basic role is played by the adiabatic hypotesis by which the interaction may be switched off in the infinite time limit. It is such a possibility which allows the definition and the introduction of non-interacting fields. In the case of damped oscillator the switching off of the interaction in the infinite time limit is not possible since time evolution is intrinsically non-unitary and the adiabatic hypotesis thus fails. As a matter of fact we have seen that the set of annihilation and creation operators changes at each time and the same concept of non-interacting field thus loses meaning. We thus conclude that damping and dissipation require a non-perturbative approach and perturbation methods may be used only for local (in time and temperature variables) fluctuations.

Finally it has been recently shown[10] that the squeezed states of light entering quantum optics[11] can be identified, up to elements of the group $\mathcal{G}$ of automorphisms of $su(1,1)$, with the states of the damped quantum harmonic oscillator.

References

[1] E. Celeghini,M. Rasetti and G.Vitiello, in Proc. 2nd Workshop on *Thermal Field Theories and Their Applications*, H.Ezawa, T.Aritmitsu and Y.Hashimoto Eds. (Elsevier, Amsterdam, 1991)

[2] M. Rasetti, *Intl. J. Mod. Phys.* **13** (1973) 425

[3] J.R. Klauder and E.C. Sudarshan, *Fundamentals of Quantum Optics* (Benjamin, New York, 1968)
[4] G. Lindblad and B. Nagel, *Ann. Inst. H. Poincaré* **XIII A** (1970) 27
[5] E.M. Stein and A.W. Knapp, *Ann. Math.* **93** (1971) 489
[6] S. De Filippo and G. Vitiello, *Lett. Nuovo Cimento* **19** (1977) 92
[7] Y. Takahashi and H. Umezawa, *Collective Phenomena* **2** (1975) 55
[8] I. Hardman, H. Umezawa and Y. Yamanaka, *Thermal Energy in QFT*, Edmonton University preprint (1990)
[9] E. Celeghini, M. Rasetti and G. Vitiello *Florence University preprint* DFF 135/5/'91
[10] E. Celeghini, M. Rasetti, M. Tarlini and G. Vitiello, *Mod. Phys. Lett.* **3 B** (1989) 1213
[11] D. Stoler, *Phys. Rev.* **D 1** (1970) 3217