A

Appendix: Optimal Control of Multivariable Systems

In the following, some basics of controller design of linear multivariable systems are recapitulated without giving derivations. Details can be found in Bryson (2002), Williams II and Lawrence (2007) and Heimann et al. (2007).

A.1 Mathematical Model

The mathematical model of an uncontrolled, time-invariant system Σ reads

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{B}\boldsymbol{u}(t) \ , \quad \boldsymbol{x}(0) = \boldsymbol{x}_0 \ , \tag{A.1}$$

$$\boldsymbol{y}(t) = \boldsymbol{C}\boldsymbol{x}(t) \tag{A.2}$$

where the vectors and matrices are defined as

$\boldsymbol{u}$	$r \times 1$ control vector,	$\boldsymbol{A}$	$n \times n$ system matrix,
$\boldsymbol{x}$	$n \times 1$ state vector,	$\boldsymbol{B}$	$n \times r$ input or control matrix,
$\boldsymbol{y}$	$m \times 1$ measurement vector,	$\boldsymbol{C}$	$m \times n$ output or measurement matrix.

Figure A.1 shows the system structure. The equations of the dynamical system (A.1) and the measurement system (A.2) can be read from the summing-point of the integrator and the system output, respectively.

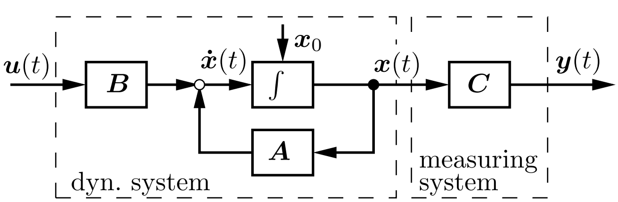

Fig. A.1. System structure

A.2 Task Formulation and Structure Issues

The aim of designing the control is to find a control law $\boldsymbol{u}(t)$ such that the system state $\boldsymbol{x}(t)$ takes a desired state $\boldsymbol{x}_s = \text{const}$ (fix point control) or $\boldsymbol{x}_s = \boldsymbol{x}_s(t)$ (follower control) keeping it regardless of the kind, position and magnitude of disturbances. Two structural questions are to be addressed before designing the control law:

I. Can the dynamical behavior of the system be changed by a control function $\boldsymbol{u}(t)$ in the desired way (is the system controllable)?
II. Can sufficient information on the system be obtained from the measurements (is the system observable)?

I. Controllability
Definition: The system Σ of order n is complete controllable, if there exists for any initial condition $\boldsymbol{x}(0) = \boldsymbol{x}_0$ and for any arbitrary state $\boldsymbol{x}_1$ a finite time $t_1 > 0$ and a input function $\boldsymbol{u}(t)$ defined in the time interval $[0, t_1]$, so that the trajectory starting in $\boldsymbol{x}_0$ reaches $\boldsymbol{x}_1$ at $t = t_1$.

Kalman - criterion:

Σ completely controllable requires

$$\text{rank}\, \boldsymbol{Q}_S = \text{rank}\, [\boldsymbol{B} \vdots \boldsymbol{A}\boldsymbol{B} \vdots \boldsymbol{A}^2\boldsymbol{B} \vdots \cdots \vdots \boldsymbol{A}^{n-1}\boldsymbol{B}] = n \; . \tag{A.3}$$

Hautus - criterion:

Σ completely controllable requires

$$(\lambda_i \boldsymbol{E} - \boldsymbol{A}^{\mathrm{T}})\bar{\boldsymbol{x}}_i = \boldsymbol{0} \;\Rightarrow \boldsymbol{B}^{\mathrm{T}}\bar{\boldsymbol{x}}_i \neq \boldsymbol{0} \; , \quad i = 1(1)n \; . \tag{A.4}$$

II. Observability
Definition: The system Σ is completely observable, if there exists for any arbitrary initial condition $\boldsymbol{x}(0) = \boldsymbol{x}_0$ a finite time $t_1 > 0$ so that the initial condition $\boldsymbol{x}_0$ can be deduced by the knowledge of the control function $\boldsymbol{u}(t)$ and the measured function $\boldsymbol{y}(t)$ in the time interval $[0, t_1]$.

Kalman - criterion:

Σ completely observable requires

$$\text{rank}\, \boldsymbol{Q}_B = \text{rank}\, [\boldsymbol{C}^{\mathrm{T}} \vdots \boldsymbol{A}^{\mathrm{T}}\boldsymbol{C}^{\mathrm{T}} \vdots \boldsymbol{A}^{T2}\boldsymbol{C}^{\mathrm{T}} \vdots \cdots \vdots \boldsymbol{A}^{Tn-1}\boldsymbol{C}^{\mathrm{T}}] = n \; . \tag{A.5}$$

Hautus - criterion:

Σ completely observable requires

$$(\lambda_i \boldsymbol{E} - \boldsymbol{A})\bar{\boldsymbol{x}}_i = \boldsymbol{0} \;\Rightarrow \boldsymbol{C}\bar{\boldsymbol{x}}_i \neq \boldsymbol{0} \; , \quad i = 1(1)n \; . \tag{A.6}$$

The Kalman criteria enable a yes/no statement for controllability and observability from the examination of the rank of the controllability matrix $\boldsymbol{Q}_S$ resp. the observability matrix $\boldsymbol{Q}_B$. The Hautus criteria enable additional statements about not controllable and not observable modes, for which $\boldsymbol{B}^{\boldsymbol{T}}\bar{\boldsymbol{x}}_{\boldsymbol{i}} = \boldsymbol{0}$ and $\boldsymbol{C}^{\boldsymbol{T}}\bar{\boldsymbol{x}}_{\boldsymbol{i}} = \boldsymbol{0}$ respectively holds. Here, $\bar{\boldsymbol{x}}_{\boldsymbol{i}}$ are the eigenvectors of the eigenvalues λ_i of system Σ.

A.3 Structure and Properties of Controllers

In the following, linear controllers for linear systems Σ are designed which transfer the state vector $\boldsymbol{x}(t)$ from the initial condition $\boldsymbol{x}(t_0) = \boldsymbol{x}_0 \neq \mathbf{0}$ (initial disturbance) to the target state $\boldsymbol{x}_s = \mathbf{0}$. This control task corresponds to a complete fixed-point control. Other control tasks can be reduced to or derived by this basic task.

The controller must assure that the controlled system

a) is asymptotically stable,
b) has a certain performance.

This aim can be achieved with the principle of feedback. The deviation of the system output from the target state is countersteered by amplifying and changing its algebraic sign and feeding it back to the system input. This is termed a linear state or output feedback respectively,

$$\boldsymbol{u}(t) = -\boldsymbol{K}_x \boldsymbol{x}(t) , \tag{A.7}$$

$$\boldsymbol{u}(t) = -\boldsymbol{K}_y \boldsymbol{y}(t) , \tag{A.8}$$

depending on the linear feedback of the state vector or the output vector, cp. Fig. A.2 (a) and (b). The constant feedback matrices $\boldsymbol{K}_x$ and $\boldsymbol{K}_y$ are composed by the control gains. The system state $\boldsymbol{x}(t)$ is often not directly available. Therefore, a state estimation $\hat{\boldsymbol{x}}(t)$ must be deduced from measurements $\boldsymbol{y}(t)$ by an observer. Normally a complete state feedback cannot be realized, but a feedback of the state estimation $\hat{\boldsymbol{x}}(t)$, Fig. A.2 c). Firstly the design of the controller is treated followed by the observer design.

A.4 Controller Design

Two methods are discussed to determine the gain matrix $\boldsymbol{K}_x$ for an ideal state feedback. These methods show the power of the feedback principle. The case of an output feedback can be realized with these methods as well, if the measurement matrix $\boldsymbol{C}$ is a regular $n \times n$ matrix. Therefore, the number of measured quantities must be equal to the number of state variables. In this case the feedback matrix reads $\boldsymbol{K}_y = \boldsymbol{K}_x \boldsymbol{C}^{-1}$. If the number of measured quantities is less then the number of state variables, more specific methods of control theory may be used to determine $\boldsymbol{K}_y$.

A.4.1 Controller Design by Pole Assignment

If and only if the system Σ of order n is complete controllable, a state feedback $\boldsymbol{u}(t) = -\boldsymbol{K}_x \boldsymbol{x}(t)$ can be found so that the closed-loop system

$$\dot{\boldsymbol{x}}(t) = \hat{\boldsymbol{A}} \boldsymbol{x}(t) , \quad \hat{\boldsymbol{A}} = \boldsymbol{A} - \boldsymbol{B} \boldsymbol{K}_x \tag{A.9}$$

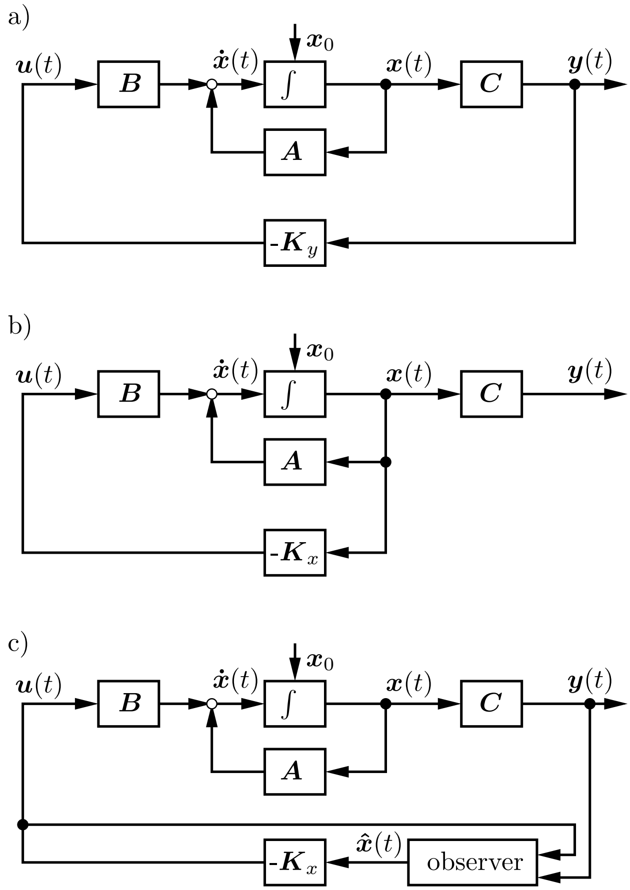

Fig. A.2. Control loops: a) output feedback; b) ideal state feedback; c) state feedback with observer

has arbitrarily prescribed eigenvalues. For complete controllable systems with only one input, $r = 1$,

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{b}u(t)\ , \quad u(t) = -\boldsymbol{k}_x^{\mathrm{T}}\boldsymbol{x}(t) \tag{A.10}$$

and the target eigenvalues $\hat{\lambda}_i, i = 1\,(1)\,n$, or the target characteristic polynomial, respectively,

$$\hat{p}(\lambda) = (\lambda - \hat{\lambda}_1)(\lambda - \hat{\lambda}_2)\ldots(\lambda - \hat{\lambda}_n) = \lambda^n + \hat{a}_1\lambda^{n-1} + \ldots + \hat{a}_{n-1}\lambda + \hat{a}_n \tag{A.11}$$

the feedback vector $\boldsymbol{k}_x^{\mathrm{T}}$ is determined uniquely,

$$\boldsymbol{k}_x^{\mathrm{T}} = \boldsymbol{e}_n^{\mathrm{T}} \boldsymbol{Q}_S^{-1} \hat{p}(\boldsymbol{A}) \; . \tag{A.12}$$

Here $\boldsymbol{e}_n$ is the n-th unit vector, $\boldsymbol{Q}_S$ the controllability matrix and $\hat{p}\,(\boldsymbol{A})$ the matrix-polynomial built of the system matrix $\boldsymbol{A}$,

$$\boldsymbol{e}_n^{\mathrm{T}} = [0, 0, 0, \ldots, 0, 1] \; , \tag{A.13}$$

$$\boldsymbol{Q}_S = [\boldsymbol{b} \vdots \boldsymbol{A}\boldsymbol{b} \vdots \cdots \vdots \boldsymbol{A}^{n-1}\boldsymbol{b}] \; , \tag{A.14}$$

$$\hat{p}(\boldsymbol{A}) = \boldsymbol{A}^n + \hat{a}_1 \boldsymbol{A}^{n-1} + \ldots + \hat{a}_{n-1}\boldsymbol{A} + \hat{a}_n \boldsymbol{E} \; . \tag{A.15}$$

For complete controllable systems with multiple inputs ambiguous solutions appear in the calculation of the feedback matrix $\boldsymbol{K}_x$. The target eigenvalues have at least to guarantee asymptotic stability, see Sect. 7.2.1.

A.4.2 Optimal Controller Due to a Quadratic Integral Criterion

For the completely controllable system Σ of order n the following cost function is used:

$$J[\boldsymbol{x}(t), \boldsymbol{u}(t)] = \frac{1}{2} \int_0^\infty [\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{Q}\boldsymbol{x}(t) + \boldsymbol{u}^{\mathrm{T}}(t)\boldsymbol{R}\boldsymbol{u}(t)\mathrm{d}t \to \mathrm{Min} \tag{A.16}$$

with the weighting matrices $\boldsymbol{Q} = \boldsymbol{Q}^{\mathrm{T}} \geq \boldsymbol{0}, \boldsymbol{R} = \boldsymbol{R}^{\mathrm{T}} > \boldsymbol{0}$. The matrices $\boldsymbol{A}$ and $\boldsymbol{Q}$ must be completely observable $\left(\mathrm{rank}\left[\boldsymbol{Q} \vdots \boldsymbol{A}^{\mathrm{T}}\boldsymbol{Q} \vdots \cdots \vdots \boldsymbol{A}^{\boldsymbol{Tn-1}}\boldsymbol{Q}\right] = n\right)$. Then a unique optimal control

$$\dot{\boldsymbol{u}}(t) = -\boldsymbol{R}^{-1}\boldsymbol{B}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{x}(t) = -\boldsymbol{K}_x\boldsymbol{x}(t) \; , \tag{A.17}$$

exists for the cost function shown above. Herein $\boldsymbol{P} = \boldsymbol{P}^{\mathrm{T}} > \boldsymbol{0}$ is the unique, positive definite solution of the Riccati equation

$$\boldsymbol{A}^{\mathrm{T}}\boldsymbol{P} + \boldsymbol{P}\boldsymbol{A} - \boldsymbol{P}\boldsymbol{B}\boldsymbol{R}^{-1}\boldsymbol{B}^{\mathrm{T}}\boldsymbol{P} + \boldsymbol{Q} = \boldsymbol{0} \; . \tag{A.18}$$

The minimal value of the cost function concludes in

$$J^* = \boldsymbol{x}_0^{\mathrm{T}}\boldsymbol{P}\boldsymbol{x}_0 \; . \tag{A.19}$$

The closed-loop control

$$\dot{\boldsymbol{x}}(t) = \hat{\boldsymbol{A}}\boldsymbol{x}(t) \; , \quad \hat{\boldsymbol{A}} = \boldsymbol{A} - \boldsymbol{B}\boldsymbol{K}_x = \boldsymbol{A} - \boldsymbol{B}\boldsymbol{R}^{-1}\boldsymbol{B}^{\mathrm{T}}\boldsymbol{P} \; , \tag{A.20}$$

is asymptotically stable.

A.4.3 Choice of Poles and Weighting Matrices

The difficulty of both methods for controller design is the appropriate assignment of the poles and the weighting matrices, respectively. It makes sense to choose the weighting matrices as diagonal matrices. The stronger the weighting of a quantity in the cost function, the smaller this quantity will be due to the optimization. Only the weighting ratio of the control variables and the state variables is important. Therefore, $\boldsymbol{R} = \boldsymbol{E}, \boldsymbol{Q} = \mathbf{diag}\,(q_i)$ can be set, where

$$q_i = \frac{u_{max}^2}{(x_{i,max})^2} \tag{A.21}$$

holds as a rule of thumb. Here $u_{max}^2 = \left(\boldsymbol{u}^{\mathrm{T}}\boldsymbol{u}\right)_{max}$ is a measure of the maximum available control energy and $x_{i,max}$ the maximum tolerated value of the i-th state variable.

As a reference for the choice of the poles, the pole configuration for two special cases of cost functions for a scalar control $u\,(r = 1)$ will be given.

a) Mirroring of the uncontrolled system poles. The special case

$$J = \lim_{\rho \to 0} \frac{1}{2} \int_0^\infty \mathrm{e}^{2\gamma t}(\rho \boldsymbol{x}^{\mathrm{T}} \boldsymbol{Q} \boldsymbol{x} + u^2)\mathrm{d}t \to \mathrm{Min} \tag{A.22}$$

results in a pole configuration that yields minimal control energy for a target stability measure γ. Let $\lambda_i, i = 1\,(1)\,n$ be the poles of the uncontrolled system Σ, $k < n$ of them on the left side and the remaining $n - k$ poles on the right side of the line $\mathrm{Re}\lambda = -\gamma$ in the root locus plane. The k poles on the left side of the line $\mathrm{Re}\lambda = -\gamma$ and the $n - k$ control path poles that are mirrored on the line are chosen as poles $\hat{\lambda}_i, i = 1\,(1)\,n$ of the closed-loop control, Litz and Preuss (1977):

$$\begin{aligned} \hat{\lambda}_i &= \lambda_i & & i = 1(1)k\,, \\ & & \text{for} & \\ \hat{\lambda}_i &= -\lambda_i - 2\gamma & & i = k+1(1)n\,. \end{aligned} \tag{A.23}$$

b) Butterworth configuration. The limit case

$$J = \lim_{\rho \to \infty} \frac{1}{2} \int_0^\infty (\rho \boldsymbol{x}^{\mathrm{T}} \boldsymbol{Q} \boldsymbol{x} + u^2)\mathrm{d}t \to \mathrm{Min} \tag{A.24}$$

results in a pole configuration that yields minimal deviation for the controlled system. Using the nomenclature defined in Sect. A.4.1 the following applies for the poles $\hat{\lambda}_i, i = 1(1)n$:

1. All poles have negative real parts.
2. The $k < n$ dominant poles satisfy the equation

$$p(\hat{\lambda}^2) = \boldsymbol{b}^{\mathrm{T}}[\mathrm{adj}(-\hat{\lambda}\boldsymbol{E} - \boldsymbol{A})]^{\mathrm{T}}\boldsymbol{Q}\ \mathrm{adj}(\hat{\lambda}\boldsymbol{E} - \boldsymbol{A})\boldsymbol{b} = 0\ . \tag{A.25}$$

3. The $n - k$ remaining poles are infinitely large and represent a stable Butterworth configuration. They are located in the root locus plane on a circle around the origin with a radius proportional to $\rho^{\frac{n-m}{2}}$ $(\rho \to \infty)$,

$$\hat{\lambda}_i = \lim_{\rho\to\infty}\left[\rho\frac{n-m}{2}\mathrm{e}^{j\psi i}\right]\ , \quad i = k+1(1)n\ , \quad j = \sqrt{-1}\ . \tag{A.26}$$

The phase angles Ψ_i follow from

$$\begin{aligned} \Psi_i &= \frac{2m+1}{n-k}\cdot 90^\circ & & n-k \text{ even}, & & \\ & & \text{for} & & m = 0, 1, 2, \dots\ , \\ \Psi_i &= \frac{2m+1}{n-k}\cdot 180^\circ & & n-k \text{ uneven}, & & \end{aligned} \tag{A.27}$$

where Ψ_i has to be chosen so that $90^\circ < \Psi_i < 270^\circ$ holds. Explicitly, for $n - m = 1\,(1)\,4$ the results are given in Table A.1. In order to get restricted control quantities and finite actuating energy, the dominant poles are set exactly, the remaining poles approximatively in Butterworth configuration, but with a finite radius.

Table A.1. Results of Eq. (A.27)

$n-k$	1	2	3	4
Ψ_i	$+180^\circ$	$\pm135^\circ$	$\pm120^\circ; +180^\circ$	$\pm112.5^\circ; \pm157.5^\circ$

A.5 Structure and Properties of Observers

The control laws presented are depending on the state vector $\boldsymbol{x}$, e. g. the optimal control (A.17), but from the measurements only the vector $\boldsymbol{y} = \boldsymbol{C}\boldsymbol{x}$ is available. In many cases the number of measures is smaller than the system order. Then the inversion of the measurement matrix $\boldsymbol{C}$ and a direct representation of the state vector by $\boldsymbol{x} = \boldsymbol{C}^{-1}\boldsymbol{y}$ is not possible. The not measured state variables $\boldsymbol{T}\boldsymbol{x}$ ($\boldsymbol{T}$ row-regular $s \times n$-matrix) can be simulated by the $s \times 1$-vector $\boldsymbol{\xi}\,(t)$ with an observer. The observer is formulated mathematically as an asymptotical estimator for the estimations $\hat{\boldsymbol{x}}\,(t)$ of the state $\boldsymbol{x}\,(t)$, Luenberger (1964).

Estimation:

$$\hat{\boldsymbol{x}}(t) = \boldsymbol{S}_1\boldsymbol{\xi}(t) + \boldsymbol{S}_2\boldsymbol{y}(t) : \lim_{t\to\infty}[\boldsymbol{x}(t) - \hat{\boldsymbol{x}}(t)] = \boldsymbol{0} \,. \tag{A.28}$$

Observation:

$$\dot{\boldsymbol{\xi}}(t) = \boldsymbol{D}\boldsymbol{\xi}(t) + \boldsymbol{T}\boldsymbol{B}\boldsymbol{u}(t) + \boldsymbol{L}\boldsymbol{y}(t) : \lim_{t\to\infty}[\boldsymbol{\xi}(t) - \boldsymbol{T}\boldsymbol{x}(t)] = \boldsymbol{0} \,. \tag{A.29}$$

With Eqs. (A.1) and (A.2) the following matrix relations are available:

$$\boldsymbol{S}_1\boldsymbol{T} + \boldsymbol{S}_2\boldsymbol{C} = \boldsymbol{E}_n \,, \tag{A.30}$$

$$\boldsymbol{D}\boldsymbol{T} - \boldsymbol{T}\boldsymbol{A} = -\boldsymbol{L}\boldsymbol{C} \,, \tag{A.31}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}(\boldsymbol{\xi} - \boldsymbol{T}\boldsymbol{x}) = \boldsymbol{D}(\boldsymbol{\xi} - \boldsymbol{T}\boldsymbol{x}) \,, \tag{A.32}$$

with

$$\mathrm{Re}\bar{\lambda}_i(\boldsymbol{D}) < 0 \,, \quad i = 1(1)s \,, \quad n - m \le s \le n \,. \tag{A.33}$$

Regarding the dimension s there are two special cases,

a) the minimal observer with $s = n - m$,
b) the complete observer with $s = n$.

For the complete observer, $\boldsymbol{T} = \boldsymbol{E}_n$ applies. From Eqs. (A.30) and (A.31) it follows

$$\boldsymbol{S}_1 + \boldsymbol{S}_2\boldsymbol{C} = \boldsymbol{E}_n \,, \quad \boldsymbol{D} = \boldsymbol{A} - \boldsymbol{L}\boldsymbol{C} \,. \tag{A.34}$$

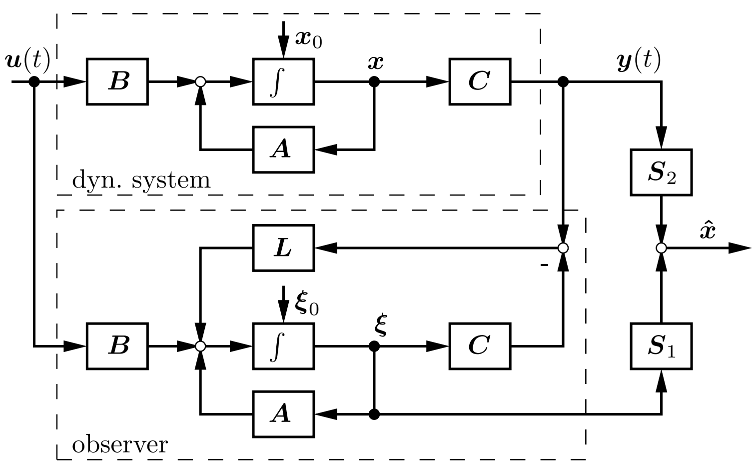

Fig. A.3. Block diagram of a dynamical system with complete observer

Therefore, the observer equations read

$$\hat{\boldsymbol{x}} = \boldsymbol{S}_1\boldsymbol{\xi} + \boldsymbol{S}_2\boldsymbol{y} \,,$$

$$\dot{\boldsymbol{\xi}} = \underbrace{(\boldsymbol{A} - \boldsymbol{L}\boldsymbol{C})}_{\boldsymbol{D}}\boldsymbol{\xi} + \boldsymbol{B}\boldsymbol{u} + \boldsymbol{L}\boldsymbol{y} \,. \tag{A.35}$$

For the error $\boldsymbol{\delta} = \boldsymbol{\xi} - \boldsymbol{x}$ it remains, cp. Eq. (A.32),

$$\dot{\boldsymbol{\delta}} = \boldsymbol{D}\boldsymbol{\delta} = (\boldsymbol{A} - \boldsymbol{L}\boldsymbol{C})\boldsymbol{\delta} \,. \tag{A.36}$$

The corresponding system setup is given in Fig. A.3. Obviously, the complete observer is basically a simulation of the original system. The error signal between original and simulated system is fed back and used for control.

A.6 Observer Design

For the design of an n-dimensional observer the gain matrix $\boldsymbol{L}$ is required, cp. Eq. (A.35). This is done by specifications of the observer matrix $\boldsymbol{D} = \boldsymbol{A} - \boldsymbol{L}\boldsymbol{C}$. Beside the requirement of asymptotical stability Eq. (A.33) the transient response of the observer shall be much faster than that of the dynamical system. With the transition to the transposed matrix $\boldsymbol{D}^{\mathrm{T}} = \boldsymbol{A}^{\mathrm{T}} - \boldsymbol{C}^{\mathrm{T}}\boldsymbol{L}^{\mathrm{T}}$ the similarity to a linear control loop is evident, cp. Eq. (A.9) with

$$\boldsymbol{A} \mathrel{\widehat{=}} \boldsymbol{A}^{\mathrm{T}}, \quad \boldsymbol{B} \mathrel{\widehat{=}} \boldsymbol{C}^{\mathrm{T}}, \quad \boldsymbol{K}_x \mathrel{\widehat{=}} \boldsymbol{L}^{\mathrm{T}} \,. \tag{A.37}$$

Therefore, the methods described in Sect. A.4 for designing linear state controller are adaptable to the design of observer, too.

A.6.1 Observer Design with Pole Assignment

If the system Σ of order n with $m \le n$ measured variables is complete observable, there exist always matrices $\boldsymbol{S}_1$, $\boldsymbol{S}_2$, $\boldsymbol{T}$, $\boldsymbol{D}$ and $\boldsymbol{L}$, which fulfill the relations of Eq. (A.30)-(A.32). The order s of the observer can be chosen arbitrarily in the range $n - m \le s \le n$. The eigenvalues $\bar{\lambda}_i\,(\boldsymbol{D})\,, i = 1\,(1)\,s$, can also be set arbitrarily regarding Eq. (A.33). For complete observable systems with only one output, $m = 1$,

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{B}\boldsymbol{u}(t) \,, \quad y(t) = \boldsymbol{c}^{\mathrm{T}}\boldsymbol{x}(t) \,, \tag{A.38}$$

the dimension s of the observer is either $s = n$ or $s = n - m = n - 1$. For an n-dimensional observer the gain matrix $\boldsymbol{L}$ results in a $n \times 1$ feedback vector $\boldsymbol{l}$. The matrix

$$\boldsymbol{D} = \boldsymbol{A} - \boldsymbol{l}\boldsymbol{c}^{\mathrm{T}} \tag{A.39}$$

has arbitrarily prescribed eigenvalues $\bar{\lambda}_i, \mathrm{Re}\bar{\lambda}_i < 0,\ i = 1\,(1)\,n$, or the corresponding characteristic polynomial

$$\bar{p}(\lambda) = (\lambda - \bar{\lambda}_1)(\lambda - \bar{\lambda}_2)\ldots(\lambda - \bar{\lambda}_n) = \lambda^n + d_1\lambda^{n-1} + \ldots + d_{n-1}\lambda + d_n\ . \tag{A.40}$$

The feedback vector $\boldsymbol{l}$ is determined uniquely as

$$\boldsymbol{l} = \bar{p}(\boldsymbol{A})(\boldsymbol{Q}_B^{\mathrm{T}})^{-1}\boldsymbol{e}_n\ . \tag{A.41}$$

Here $\boldsymbol{e}_n$ is the n-th unit vector, $\boldsymbol{Q}_B$ the observability matrix and $\bar{p}\,(\boldsymbol{A})$ the matrix-polynomial built of the system matrix $\boldsymbol{A}$:

$$\boldsymbol{e}_n = [0, 0, 0 \ldots 0, 1]^{\mathrm{T}}\ , \tag{A.42}$$

$$\boldsymbol{Q}_B = [\boldsymbol{c} \vdots \boldsymbol{A}^{\mathrm{T}}\boldsymbol{c} \vdots \cdots \vdots \boldsymbol{A}^{Tn-1}\boldsymbol{c}]\ , \tag{A.43}$$

$$\bar{p}(\boldsymbol{A}) = \boldsymbol{A}^n + d_1\boldsymbol{A}^{n-1} + \ldots + d_{n-1}\boldsymbol{A} + d_n\boldsymbol{E}\ . \tag{A.44}$$

The matrices $\boldsymbol{S_1}$ and $\boldsymbol{S_2}$ can be chosen as

$$\boldsymbol{S}_1 = \boldsymbol{E}_n - \frac{\boldsymbol{c}\boldsymbol{c}^{\mathrm{T}}}{\boldsymbol{c}^{\mathrm{T}}\boldsymbol{c}}\ , \quad \boldsymbol{S}_2 = \frac{\boldsymbol{c}}{\boldsymbol{c}^{\mathrm{T}}\boldsymbol{c}} \tag{A.45}$$

or

$$\boldsymbol{S}_1 = \boldsymbol{E}_n\ , \quad \boldsymbol{S}_2 = \boldsymbol{0}\ . \tag{A.46}$$

A.6.2 Optimal Observer Due to a Quadratic Integral Criterion

For an n-dimensional observer the equation for the error $\boldsymbol{\delta} = \boldsymbol{\xi} - \boldsymbol{x}$ is given by Eq. (A.36). The transposed error differential equation

$$\dot{\boldsymbol{\eta}}(t) = (\boldsymbol{A}^{\mathrm{T}} - \boldsymbol{C}^{\mathrm{T}}\boldsymbol{L}^{\mathrm{T}})\boldsymbol{\eta}(t) \tag{A.47}$$

has the structure of a control loop with state feedback, cp. Eq. (A.9),

$$\dot{\boldsymbol{\eta}}(t) = \boldsymbol{A}^{\mathrm{T}}\boldsymbol{\eta}(t) + \boldsymbol{C}^{\mathrm{T}}\boldsymbol{\nu}(t)\ , \quad \boldsymbol{\eta}(0) = \boldsymbol{\eta}_0\ , \tag{A.48}$$

$$\boldsymbol{\nu}(t) = -\boldsymbol{L}^{\mathrm{T}}\boldsymbol{\eta}(t)\ . \tag{A.49}$$

The complete controllability of the system of Eq. (A.48) corresponds to the complete observability of the system given by Eqs. (A.1) and (A.2). In analogy to Sect. A.4.2 the following can be stated: For the complete observable system or the complete controllable system (A.48), respectively, both of order n, and the cost function

$$J[\boldsymbol{\eta}(t), \boldsymbol{\nu}(t)] = \frac{1}{2}\int_0^\infty [\boldsymbol{\eta}^{\mathrm{T}}(t)\boldsymbol{Q}\boldsymbol{\eta}(t) + \boldsymbol{\nu}^{\mathrm{T}}(t)\boldsymbol{R}\boldsymbol{\nu}(t)]\mathrm{d}t \rightarrow \mathrm{Min} \tag{A.50}$$

with the weighting matrices $\boldsymbol{Q} = \boldsymbol{Q}^{\mathrm{T}} \geq \mathbf{0}, \boldsymbol{R} = \boldsymbol{R}^{\boldsymbol{T}} > \mathbf{0}$ and $\left(\boldsymbol{A}^{\boldsymbol{T}}\boldsymbol{Q}\right)$ completely observable, there exists one unique optimal control

$$\boldsymbol{\nu}^{*}(t) = -\boldsymbol{R}^{-1}\boldsymbol{C}\boldsymbol{P}\boldsymbol{\eta}(t) = -\boldsymbol{L}^{\mathrm{T}}\boldsymbol{\eta}(t) \ . \tag{A.51}$$

Herein $\boldsymbol{P} = \boldsymbol{P}^{\mathrm{T}} > \mathbf{0}$ is the unique, positive definite solution of the Riccati equation

$$\boldsymbol{A}\boldsymbol{P} + \boldsymbol{P}\boldsymbol{A}^{\mathrm{T}} - \boldsymbol{P}\boldsymbol{C}^{\mathrm{T}}\boldsymbol{R}^{-1}\boldsymbol{C}\boldsymbol{P} + \boldsymbol{Q} = \mathbf{0} \ . \tag{A.52}$$

The optimal value for the criterion results in

$$J^{*} = \boldsymbol{\eta}_0^{\mathrm{T}}\boldsymbol{P}\boldsymbol{\eta}_0 \ . \tag{A.53}$$

The closed-loop control

$$\dot{\boldsymbol{\eta}}(t) = \boldsymbol{D}^{\mathrm{T}}\boldsymbol{\eta}(t) \ , \quad \boldsymbol{D}^{\mathrm{T}} = \boldsymbol{A}^{\mathrm{T}} - \boldsymbol{C}^{\mathrm{T}}\boldsymbol{L}^{\mathrm{T}} = \boldsymbol{A}^{\mathrm{T}} - \boldsymbol{C}^{\mathrm{T}}\boldsymbol{R}^{-1}\boldsymbol{C}\boldsymbol{P} \tag{A.54}$$

is asymptotically stable, and, therefore, the n-dimensional observer of Eq. (A.36), too. The results of Sect. A.4.3 for the choice of the poles and the weighting matrices can also be used for designing the observer.

A.7 Structure of (Optimal) Controlled Multivariable Systems

For the complete controllable and complete observable system Σ a state feedback according to Sect. A.4 and a state observer according to Sect. A.6 can be found, each with the specified properties. The structure of the complete system is depicted in Fig. A.2 c).

The complete multivariable control system is described mathematically as:

Dynamical system:

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{B}\boldsymbol{u}(t) \ , \tag{A.55}$$

Measurement:

$$\boldsymbol{y}(t) = \boldsymbol{C}\boldsymbol{x}(t) \ ,$$

Observer:

$$\dot{\boldsymbol{\xi}}(t) = \boldsymbol{D}\boldsymbol{\xi}(t) + \boldsymbol{T}\boldsymbol{B}\boldsymbol{u}(t) + \boldsymbol{L}\boldsymbol{y}(t) \ , \tag{A.56}$$

$$\hat{\boldsymbol{x}}(t) = \boldsymbol{S}_1\boldsymbol{\xi}(t) + \boldsymbol{S}_2\boldsymbol{y}(t) \ ,$$

Controller:

$$\boldsymbol{u}(t) = -\boldsymbol{K}_x\hat{\boldsymbol{x}}(t) \ . \tag{A.57}$$

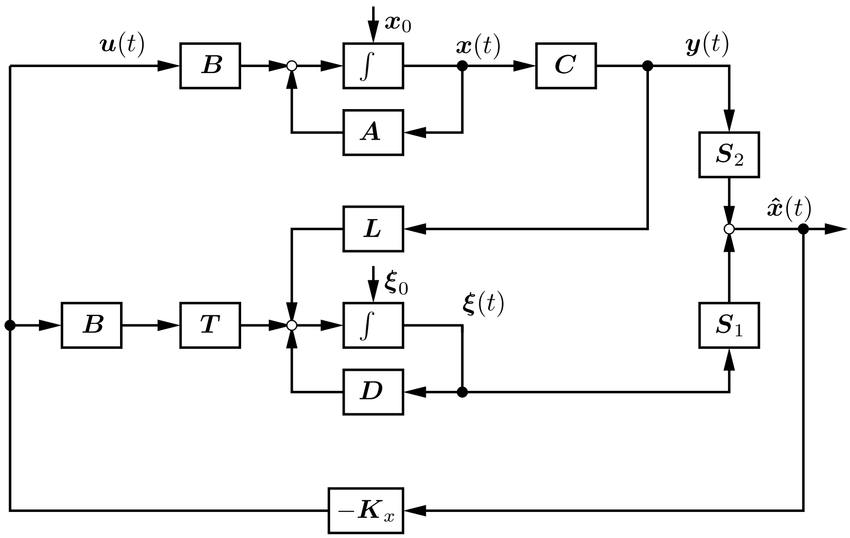

Fig. A.4. Block diagram of the complete control system

The eigenvalues of the linear, time invariant complete system, Fig. A.4, are composed of the n eigenvalues of the control loop with ideal state feedback and the s eigenvalues of the observer. For the characteristic polynomial and the eigenvalues holds:

$$p_{ges}(\lambda) = \hat{p}_{A-BK_x}(\lambda) \cdot \bar{p}_D(\lambda) \ , \tag{A.58}$$

$$\lambda_{i,ges} = \begin{cases} \hat{\lambda}_i(\boldsymbol{A} - \boldsymbol{B}\boldsymbol{K}_x) & \quad i = 1(1)n \\ & \text{for} \\ \hat{\lambda}_{i-n}(\boldsymbol{D}) & \quad i = (n+1)(1)(n+s) \ . \end{cases}$$

The realizable complete control loop is extended by an observer compared to the ideal state feedback. With respect to the eigenvalues of the complete system, the controller can be designed separately from the design of the observer.

B

Appendix: Key Words

B.1 English - German

A

acceleration	Beschleunigung
aligning torque stiffness	Bohrmomentbeiwert

B

beam	Balken
continuously bedded	kontinuierlich gebetteter
double-span	Zweifeldträger
pillared	periodisch gestützter
single-span	Einfeldträger
bicycle model	Riekert-Schunck Modell
body slip angle	Schwimmwinkel
boundary value problem	Randwertproblem
braking force	Bremskraft
brush model	Bürstenmodell

C

Cardano angle	Kardanwinkel
connecting element	Bindungselement
constraint	Bindung, Zwangsbedingung
contact force	Kontaktkraft
law	-gesetz
tire-road	Reifen-Straße
wheel-rail	Rad-Schiene
control gain	Reglerverstärkung
control vector	Steuervektor
controllability	Steuerbarkeit

controller	Regler
design	-entwurf
gain	-verstärkung
cornering	Kurvenfahrt
stiffness	Seitenkraftbeiwert
cost function	Gütekriterium
coupling element	Koppelelement
creepage	Radsatzschlupf

D

Damper	Dämpfer
characteristic	-Kennlinie
parallel combination	-Parallelschaltung
series combination	-Reihenschaltung
damping	Dämpfung
normalized coefficient	-smaß, Lehrsches
degree of efficiency	Wirkungsgrad
degree of freedom	Freiheitsgrad
degree of unevenness	Unebenheitsgrad
derailment	Entgleisung
differential	Differentialgetriebe
driving comfort	Fahrkomfort
driving performance	Fahrleistung
driving performance diagram	Fahrzustandsschaubild
driving safety	Fahrsicherheit
driving stability	Fahrstabilität
durability	Lebensdauer

E

eigenmode	Schwingungsform
elementary rotation	Elementardrehung
equation of motion	Bewegungsgleichung
equation of reaction	Reaktionsgleichung
excitation	Erregung
bump	Bodenwellen-
ramp	Rampen-
random	Zufalls-
time delay	zeitverzögerte
unbalance	Unwucht-
exposure time	Einwirkungsdauer

F

feedback matrix	Rückführmatrix
force	Kraft
applied	eingeprägte

constraint	Reaktions-
contact	Kontakt-
dissipative	dissipative
cornerning	Seiten-
friction	Reibungs-
generalized	verallgemeinerte
lateral	Seiten-
reaction	Reaktions-
travelling	bewegte
force actuator	Kraftstellglied
foundation	Bettung
frame	Koordinatensystem
frequency decoupling	Frequenzentkopplung
frequency response	Frequenzgang
friction	Reibung
sliding	Gleit-
sticking	Haft-
friction coefficient	Reibungsbeiwert

G

gradability	Steigfähigkeit
gravitational stiffness	Gravitationssteifigkeit
guidance system	Führsystem
guideway	Fahrweg
gyro matrix	Kreiselmatrix

H

handling	Kurshaltung
hunting motion	Sinuslauf

I

inertia matrix	Massenmatrix
inertia properties	Trägheitseigenschaften
initial value problem	Anfangswertproblem
input matrix	Eingangsmatrix

L

lateral motion	Querbewegung

M

maglev vehicle	Magnetschwebefahrzeug
magnetic actuator	Magnetstellglied
magnetic wheel	Magnetisches Rad
maximum rolling contact coefficient	Kraftschlußbeiwert
mean value	Mittelwert

mileage	Kraftstoffverbrauch
model	Ersatzsystem
modeling	Modellbildung
moment of inertia	Trägheitsmoment
motion	Bewegung
hunting	Sinuslauf
plane	ebene
moving load	bewegte Last
multibody system	Mehrkörpersystem
multivariable system	Mehrgrößensystem

N

node displacement	Knotenverschiebung
noise process	Rauschprozeß

O

observability	Beobachtbarkeit
oversteer	übersteuern

P

perception	Wahrnehmung
pitch	nicken
pole assignment	Polvorgabe
position vector	Lagevektor
power spectral density	spektrale Leistungsdichte
probability density	Wahrscheinlichkeitsdichte
product of inertia	Devitationsmoment

R

railway wheelset	Eisenbahnradsatz
random vibration	Zufallsschwingungen
reliability interval	Vertrauensintervall
resistance	Widerstand
climbing	Steigungs-
rolling	Roll-
ride comfort	Fahrkomfort
rolling condition	Rollbedingung
rolling contact coefficient	Kraftschlußbeanspruchung
normalized	normierte
maximum	maximale
rolling elastic contact	rollender elastischer Kontakt
rolling radius	Rollradius
roll	rollen
rotation matrix	Drehmatrix

S

safety margin	Sicherheitsreserve
saturation	Sättigung
shape filter	Formfilter
slip	Schlupf
aligning	Bohr-
braking	Brems-
driving	Antriebs-
lateral	Quer-
longitudinal	Längs-
micro	Mikro-
rigid body	Starrkörper-
slip angle	Schräglaufwinkel
speed	Fahrgeschwindigkeit
spring	Feder
characteristic	-kennlinie
leaf	Blatt-
parallel connection	-Parallelschaltung
series connection	-Reihenschaltung
state equation	Zustandsgleichung
stiffness matrix	Steifigkeitsmatrix
subsystem	Teilsystem
surface pressure	Flächenpressung
suspension	Federung
suspension travel	Federweg
system boundary	Systemgrenze

T

time integration	Zeitintegration
tire	Reifen
carcass	-karkasse
torque	Moment
aligning	Bohr-
braking	Brems-
driving	Antriebs-
track model	Schienenmodell
trail	Nachlauf
suspension	konstruktiver
trailer	Anhänger
transmission ratio	Übersetzung
twist	Bewegungswinder

U

understeer	untersteuern
unevenness	Unebenheit

V

vehicle-guideway-system	Fahrzeug-Fahrweg-System
velocity	Geschwindigkeit
vibration	Schwingung

W

waviness	Welligkeit
wheel	Rad
braked	gebremstes
conical	konisches
cornering	schräglaufendes
deformable	deformierbar
driven	angetriebenes
elastic	elastisches
rigid	starres
wheel load	Radlast
wheelset	Radsatz
wrench	Kraftwinder

Y

yaw	gieren, schleudern

B.2 Deutsch - Englisch

A	
Anfangswertproblem	initial value problem
Anhänger	trailer
B	
Balken	beam
Einfeldträger	single-span
kontinuierlich gebetteter	continously bedded
periodisch gestützter	pillared
Zweifeldträger	double-span
Beiwert	coefficient
Beobachtbarkeit	observability
Beschleunigung	acceleration
Bettung	foundation
bewegte Last	moving load
Bewegung	motion
ebene	plane
Sinuslauf	hunting
Bewegungsgleichung	equation of motion
Bewegungswinder	twist
Bremskraft	braking force
Bindung	constraint
Bindungselement	connecting element
Bohrmomentbeiwert	aligning torque stiffness
Bürstenmodell	brush model
D	
Dämpfer	damper
-Parallelschaltung	parallel combination
-Reihenschaltung	series combination
-kennlinie	characteristic
Dämpfung	damping
-smaß, Lehrsches	normalized coefficient
Devitationsmoment	product of inertia
Differentialgetriebe	differential
Drehmatrix	rotation matrix
E	
Eingangsmatrix	input matrix
Einwirkungsdauer	exposure time
Eisenbahnradsatz	railway wheelset

Elementardrehung	elementary rotation
Entgleisung	derailment
Erregung	excitation
Bodenwellen-	bump
Rampen-	ramp
stochastische	stochastic
Unwucht-	unbalance
zeitverzögerte	time delay
Zufalls-	random
Ersatzsystem	model

F

Fahrgeschwindigkeit	speed
Fahrkomfort	driving comfort, ride comfort
Fahrleistung	driving performance
Fahrsicherheit	driving safety
Fahrstabilität	driving stability
Fahrweg	guideway
Fahrzeug-Fahrweg-System	vehicle-guideway-system
Fahrzustandsschaubild	driving performance diagram
Feder	spring
-kennlinie	characteristic
-Parallelschaltung	parallel connection
-Reihenschaltung	series connection
Blatt-	leaf
Federung	suspension
Federweg	suspension travel
Flächenpressung	surface pressure
Formfilter	shape filter
Freiheitsgrad	degree of freedom
Frequenzentkopplung	frequency decoupling
Frequenzgang	frequency response
Führsystem	guidance system

G

Geschwindigkeit	velocity
gieren	yaw
Gravitationssteifigkeit	gravitational stiffness
Gütekriterium	cost function

K

Kardanwinkel	Cardano angle
Knotenverschiebung	node displacement
Kontaktkraft	contact force
-gesetz	law

Rad-Schiene	wheel-rail
Reifen-Straße	tire-road
Koordinatensystem	frame
Koppelelement	coupling element
Kraft	force
bewegte	travelling
dissipative	dissipative
eingeprägte	applied
Kontakt-	contact
Reaktions-	constraint
Reibungs-	friction
Seiten-	cornerning
verallgemeinerte	generalized
Kraftschlußbeanspruchung	rolling contact coefficient
normierte	normalized
Kraftschlußbeiwert	maximum rolling contact coefficient
Kraftstellglied	force actuator
Kraftstoffverbrauch	mileage
Kraftwinder	wrench
Kreiselmatrix	gyro matrix
Kurshaltung	handling
Kurvenfahrt	cornering

L

Lagevektor	position vector
Lebensdauer	durability

M

Magnetisches Rad	magnetic wheel
Magnetschwebefahrzeug	maglev vehicle
Magnetstellglied	magnetic actuator
Massenmatrix	inertia matrix
Mehrgrößensystem	multivariable system
Mehrkörpersystem	multibody system
Mittelwert	mean value
Modellbildung	modeling
Moment	torque
Antriebs-	driving
Bohr-	aligning
Brems-	braking

N

Nachlauf	trail
konstruktiver	suspension
nicken	pitch

P

Polvorgabe	pole assignment

Q

Querbewegung	lateral motion

R

Rad	wheel
angetriebenes	driven
deformierbar	deformable
elastisches	elastic
gebremstes	braked
konisches	conical
schräglaufendes	cornering
starres	rigid
Radlast	wheel load
Radsatz	wheelset
Radsatzschlupf	creepage
Randwertproblem	boundary value problem
Rauschprozeß	noise process
Reaktionsgleichung	equation of reaction
Regler	controller
-entwurf	design
-verstärkung	gain
Reibung	friction
Gleit-	sliding
Haft-	sticking
Reibungsbeiwert	friction coefficient
Reifen	tire
-karkasse	carcass
Riekert-Schunck Modell	bicycle model
Rollbedingung	rolling condition
rollen	roll
rollender elastischer Kontakt	rolling elastic contact
Rollradius	rolling radius
Rückführmatrix	feedback matrix

S

Sättigung	saturation
Schienenmodell	track model
schleudern	yaw
Schlupf	slip
Antriebs-	driving
Bohr-	aligning

Brems-	braking
Längs-	longitudinal
Mikro-	micro
Quer-	lateral
Starrkörper-	rigid body
Schräglaufwinkel	slip angle
Schwimmwinkel	body slip angle
Schwingung	vibration
Schwingungsform	eigenmode
Seitenkraftbeiwert	cornering stiffness coefficient, lateral force coefficient
Sicherheitsreserve	safety margin
Sinuslauf	hunting motion
spektrale Leistungsdichte	power spectral density
Steigfähigkeit	gradability
Steuervektor	control vector
Steuerbarkeit	controllability
Systemgrenze	system boundary

T

Teilsystem	subsystem
Trägheitseigenschaften	inertia properties
Trägheitsmoment	moment of inertia

U

Übersetzung	transmission ratio
übersteuern	oversteer
Unebenheit	unevenness
Unebenheitsgrad	degree of unevenness
untersteuern	Understeer

V

Vertrauensintervall	reliability interval

W

Wahrnehmung	perception
Wahrscheinlichkeitsdichte	probability density
Welligkeit	waviness
Widerstand	resistance
Roll-	rolling
Steigungs-	climbing
Wirkungsgrad	degree of efficiency

Z

Zeitintegration	time integration

Zufallsschwingungen	random vibration
Zustandsgleichung	state equation
Zwangsbedingung	constraint

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