

PORTFOLIO MANAGEMENT WITH HEURISTIC OPTIMIZATION

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Portfolio Management with Heuristic Optimization

by

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Preface

Managing financial portfolios is primarily concerned with finding a combination of assets that serves an investor's needs and demands the best. This includes a wide range of aspects such as the analysis of the investor's attitude towards risk, expected return and consumption; estimations of future payoffs of the financial securities and the risk associated with it have to be made; assessing the relationships between securities; determining fair prices for these securities – and finding an optimal combination of financial securities. Many of these tasks are interrelated: what is an optimal combination depends on the investor's preferences as well as on the properties of the assets, which, in return, will affect what is considered a fair price and *vice versa*.

The usual (theoretical) frameworks for portfolio management and portfolio optimization assume markets to be frictionless. Though it drives the models away from reality, this assumption has long been considered the only way to make these models approachable. However, with the advent of a new type of optimization and search techniques, *heuristic optimization*, more complex scenarios and settings can be investigated and many of these simplifying assumptions are no longer necessary.

This book is merely concerned with problems in portfolio management when there are market frictions and when there are no ready-made solutions available. For this purpose, the first two chapters present the foundations for portfolio management and new optimization techniques. In the subsequent chapters, financial models will be enhanced by problems and aspects faced in real-life such as transaction costs, indivisible assets, limits on the number of assets, alternative risk measures and descriptions of the returns' distributions, and so on. For each of these enhanced problems, a detailed presentation of the model will be followed by a description of how it can be approached with heuristic optimization. Next, the suggested approaches will be applied to empirical studies and the conclusions for financial theory will be discussed.

Non-technical Summary

The theoretical foundation to portfolio management as we know it today was laid by Harry M. Markowitz by stating a parametric optimization model. The gist of this model is to split the portfolio selection process into two steps where first the set of optimal portfolios is determined and then the investor chooses from this set that portfolio that suits her best. Markowitz's approach therefore includes (i) measuring the expected return and risk of the available assets (independently of the investor's beliefs and preferences), and (ii) making certain assumptions about the investor's utility functions (independently of the available assets). These two steps are then brought together in a quadratic optimization problem. This model, by now the centre of *Modern Portfolio Theory*, provoked a revised notion of risk and in due course of what is a fair risk premium.

Chapter 1 presents some aspects of the financial theory underlying this contribution, including the portfolio selection problem in a Markowitz framework and selected related and follow-up literature. In addition, two equilibrium models will be presented: the *Capital Asset Pricing Model (CAPM)*, which, in this contribution, will be used to generate data for equilibrium markets, and the concurring *Arbitrage Pricing Theory (APT)* for which relevant risk factors will be identified. The chapter concludes after a short presentation of alternative approaches to portfolio management.

With all its merits, the Markowitz model has a major downside: to get a grip of the computational complexity, it has to rely on a number of rather strict technical assumptions which are more or less far from reality: markets are assumed to be perfect in the sense that there are neither taxes nor transactions costs and assets are infinitely divisible; investors make their decisions at exactly one point in time for a single-period horizon; and the means, standard deviations and correlation coefficients are sufficient to describe the assets' returns. Though there exists no closed-form solution for the Markowitz model, the simplifying assumptions allow for a solution with standard software in reasonable time if the number of assets is not too large.

The limitations of the original Markowitz framework have stimulated a number of extended or modified models. These models allow for valuable insights – yet

still have to make simplifying assumptions in order to be solvable: seemingly simple questions such as adding proportional plus minimum transactions costs, taking into account that usually stocks can be traded in whole-numbered lots, or allowing for non-parametric empirical distributed returns are unsolvable with standard methods. It therefore appears desirable to have alternative methods that can handle highly demanding optimization problems.

One way out of this dilemma is *heuristic optimization (HO)*. The techniques employed in HO are mostly general purpose search methods that do not derive the solution analytically but by iteratively searching and testing improved or modified solutions until some convergence criterion is met. Since they usually outperform traditional numerical procedures, they are well suited for empirical and computational studies. **Chapter 2** presents some general concepts and standard HO algorithms.

Having introduced some basic concepts, heuristic optimization techniques are applied to some portfolio selection problems which cannot be solved with other, more traditional methods.

The effects of magnitude of initial wealth, type of transactions costs as well as integer constraints on the portfolio selection problem will be discussed based on DAX data in **chapter 3**. We distinguish a number of cases where investors with different initial wealth face proportional costs and/or fixed transactions costs. As the associated optimization problem cannot be solved with standard optimization techniques, the literature so far has confined itself to rather simple cases; to our knowledge, there are no results for an equally comprehensive model. This problem is usually approached by first solving the problem without these aspects and then fitting the results on the real-world situation. The findings from the empirical study illustrate that this might lead to severely inferior solutions and wrong decisions: Unlike predicted by theory when the usual simplifications apply, investors are sometimes well-advised to have a rather small number of different assets in their portfolios, and the optimal weights are not directly derivable from those for frictionless markets.

For various reasons, investors tend to hold a rather small number of different assets in their portfolios. Also, it is a well-known fact that much of a portfolio's diversification can be achieved with a rather small number of assets – yet, to our knowledge there exist only rough estimates based on standard rules or simple simulations to evaluate this fact. **Chapter 4** focuses on the selection problem under cardinality

constraints, i.e., when there are explicit bounds on the number of different assets. The empirical study uses data for the DAX, FTSE and S&P 100. The main results are that small (yet well-selected) portfolios can be almost as well-diversified as large portfolios and that standard rules applied in practice can be outperformed.

Chapter 5, too, investigates the effects of cardinality constraints yet in a different setting where not just one specific portfolio, but the whole so-called “efficient sets” are to be identified. In order to meet the high computational complexity of this problem, a new algorithm is developed and tested against alternative optimization heuristics. With the focus on the computational aspects, it is shown that hybrid algorithms, combining aspects from different heuristic methods can be superior to basic algorithms and that heuristic optimization algorithms can be modified according to particular aspects in the problems. With this new algorithm at hand, the highly demanding optimization problem can now be approached.

The usual definition of “financial risk” captures the assumed (positive and negative) deviations from the expected returns. In some circumstances, however, the investor might be more interested in the maximum loss with a certain probability or the expected loss in the worst cases. Hence, alternative risk measures such as *Value at Risk (VaR)* and *Expected Shortfall (ES)* have gained considerable attention. **Chapter 6** is concerned with the question of whether these new risk measures actually make good risk constraints when the investor is interested in limiting the portfolio’s potential losses. Based on empirical studies for bond markets and stock markets, we find that VaR has severe shortcomings when it is used as an explicit risk constraint, in particular when the normality assumption of the expected returns is abandoned (as has often been demanded by theory and practice).

The *Arbitrage Pricing Theory (APT)* is sometimes considered superior to other equilibrium pricing models such as, e.g., the CAPM as it does not use an (actually unobservable) market portfolio but a set of freely selectable (and observable) factors. The major shortfall of the APT, however, is that there are no straightforward or *a priori* rules of how to find the ideal set of factors: there are not always “natural” candidates for factors, standard choices do not work equally well for all assets (or are not applicable for other reasons). Given a set of potential candidates, the associated selection problem is computationally extremely demanding. **Chapter 7** finds that this model selection problem, too, can be approached with heuristic search methods. The selected combinations of factors are likely to identify fundamentally plau-

sible indicators and they are likely to explain a considerable share of the variation in the assets' returns.

Chapter 8 concludes and presents an outlook on possible follow-up research.

The prime focus of this contribution is on individual investment decisions under market frictions. The main part of the study will therefore consider individual investors who already have estimates for future returns and risks but face the problem of how to translate these estimates into optimal portfolio selections (chapters 3 – 6) or how to translate estimates for aggregated market factors into pricing models for individual assets (chapter 7) in the first place. In all of these problems, the investors are considered to be rational and risk averse price takers operating in equilibrium markets.

This contribution therefore aims to answer financial management problems that are well identified in financial theory and faced by the investment industry but could not yet be answered satisfactorily by the literature. Due to the restrictions in traditional optimization methods, the respective models had to rely on simplifying assumptions and stylized facts that restrained the applicability of the results. In this contribution, an alternative route is chosen and new optimization methods are applied that are capable of dealing with otherwise unanswerable problems. The results show that this approach is capable of identifying shortcomings of traditional approaches, that market frictions and complex constraints can now easily and completely be incorporated in the optimization process without the usual prior simplifications (which, as will be shown, can even be misleading), and that problems can be solved for which just approximations or rules of the thumb existed so far.

The results from these studies also indicate the gain from the application of new methods such as heuristic optimization: Models and problems can be investigated that allow for more complexity and are therefore closer to reality than those approachable with traditional methods, which eventually also contributes to a better understanding of financial markets.

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