

CRM Series in Mathematical Physics

Editorial Board:

Joel S. Fieldman
Department of Mathematics
University of British Columbia
Vancouver, British Columbia V6T 1Z2
Canada
feldman@math.ubc.ca

Duong H. Phong
Department of Mathematics
Columbia University
New York, NY 10027-0029
USA
phong@math.columbia.edu

Yvan Saint-Aubin
Département de Mathématiques
et de Statistique
Université de Montréal
C.P. 6128, Succursale Centre-ville
Montréal, Québec H3C 3J7
Canada
saint@math.ias.edu

Luc Vinet
Centre de Recherches Mathématiques
Université de Montréal
C.P. 6128, Succursale Centre-ville
Montréal, Québec H3C 3J7
Canada
vinet@crm.umontreal.ca

For other titles published in this series, go to
<http://www.springer.com/series/3872>

John Harnad
Editor

Random Matrices, Random Processes and Integrable Systems

 Springer

Editor

John Harnad
Department of Mathematics and Statistics
Concordia University
1455 de Maisonneuve Blvd. West
Montréal, Québec, H3G 1M8
Canada
and
Centre de Recherches Mathématiques
Université de Montréal
C.P. 6128, Succ. Centre ville
Montréal, Québec H3C 3J7
Canada
harnad@crm.umontreal.ca

ISBN 978-1-4419-9513-1 e-ISBN 978-1-4419-9514-8
DOI 10.1007/978-1-4419-9514-8
Springer New York Dordrecht Heidelberg London

Library of Congress Control Number: 2011925846

© Springer Science+Business Media, LLC 2011

All rights reserved. This work may not be translated or copied in whole or in part without the written permission of the publisher (Springer Science+Business Media, LLC, 233 Spring Street, New York, NY 10013, USA), except for brief excerpts in connection with reviews or scholarly analysis. Use in connection with any form of information storage and retrieval, electronic adaptation, computer software, or by similar or dissimilar methodology now known or hereafter developed is forbidden.

The use in this publication of trade names, trademarks, service marks, and similar terms, even if they are not identified as such, is not to be taken as an expression of opinion as to whether or not they are subject to proprietary rights.

Printed on acid-free paper

Springer is part of Springer Science+Business Media (www.springer.com)

Preface

John Harnad

Department of Mathematics and Statistics, Concordia University,
1455 de Maisonneuve Blvd. West, Montréal, Québec, H3G 1M8, Canada
Centre de Recherches Mathématiques, Université de Montréal, C.P. 6128, Succ.
Centre ville, Montréal, Québec H3C 3J7, Canada, harnad@crm.umontreal.ca

This volume is intended as an introduction and overview of the three domains featured in the title, with emphasis on the remarkable links between them. It has its origins in an intensive series of advanced courses given by the authors at the Centre de Recherches Mathématiques in Montréal in the summer of 2005. Since then, it has grown considerably beyond the material originally presented, and been extended in content and scope. The original courses were interlaced in schedule with an extended research workshop, dealing with related topics, whose proceedings, in enhanced form, have since been published as a special issue of *Journal of Physics A* [1].

The participants at the two events included a large number of graduate students and postdoctoral fellows, as well as many of the researchers who had made pioneering contributions to these domains. The original content of the lecture series, given by several of the subject's leading contributors, has since been considerably developed and polished, to the point where it may be viewed as both a research survey and pedagogical monograph providing a panoramic view of this very rich and still rapidly developing domain.

In Part I, we have combined the introductory chapters by Pierre van Moerbeke, covering nearly all the topics occurring in the rest of the volume, together with the further, more detailed chapters linking random matrices and integrable systems, concerning mainly their joint work, written by Mark Adler.

Van Moerbeke's part consists of nine chapters. The first of these concerns random permutations, random words and percolation, linked with random partitions through the Robinson–Schensted–Knuth (RSK) correspondence. This includes an introduction to the Ulam problem (concerning the longest increasing subsequence of a random permutation), the Plancherel measure on partitions, the relation to non-intersecting random walkers, as well as queueing problems and polynuclear growth. He then discusses Toeplitz determinants,

infinite wedge product representations, and the non-positive generalized probability measure of Borodin–Okounkov–Olshanski, expressed as a matrix integral over $U(n)$. Several examples follow, including the Poissonized Plancherel measure, bringing in the use of Fredholm determinants of integral operators with a variety of kernels (Bessel, Charlier and Meixner polynomial type kernels), with applications to the distributions arising in the above random processes.

There follows a discussion of limit theorems, such as the Vershik–Kerov limiting shape of a random partition and the Tracy–Widom distribution for the longest increasing subsequences, as well as geometrically distributed percolation problems. Next, it is shown that the multiple (N -fold) integrals obtained upon reducing $U(n)$ invariant Hermitian matrix models with arbitrary exponential trace invariant series deformations are tau functions of the KP (Kadomtsev–Petviashvili) integrable hierarchy, as well as satisfying the usual Toda lattice equations for varying N 's, and the Hirota bilinear relations. Next, Virasoro algebra constraints are deduced for the multiple integrals defining the generalized β -type integrals. There is also a review of the basic finite N Hermitian matrix model results, including the form of the reduced integrals over the eigenvalues, computation of the determinantal form of the correlation functions in terms of suitable (Christoffel–Darboux) correlation kernels, and Fredholm integral expressions for the gap probabilities. The PDEs satisfied by these Fredholm integrals when the endpoints of the support intervals are varied are derived, for a variety of limiting kernels.

In the subsequent chapters, by Mark Adler, further links between random matrix theory and integrable models are developed, using vertex operator constructions. A soliton-like tau function is constructed using a Fredholm determinant and shown to satisfy Virasoro constraints. For gap probabilities, these are used as a vehicle to deduce differential equations that they must satisfy. There are also a number of lattice systems that are constructed using as phase space variables that are defined as N -fold matrix-like integrals. Exponential trace series deformations of 2-matrix integrals are shown to satisfy the equations of the 2-Toda hierarchy, and bilinear identities. Using the Virasoro constraints, PDEs for the gap probabilities are also deduced.

There follows a discussion of the Dyson diffusion process and its relation to random matrices, and chains of random matrices, as well as the bulk and edge scaling limits (sine and Airy processes). Equations are derived for these processes similar to those for the gap probabilities, with respect to the edges of the windows where the nonintersecting random paths are excluded, as well as asymptotic expansions. The GUE with external source and its relation to conditioned non-intersecting Brownian motion, as developed by Aptekarev, Bleher and Kuijlaars is developed, together with its relation to the Riemann–Hilbert problem for multiple orthogonal polynomials. (See Bleher's chapters for further details.) Finally there is a derivation of PDEs for the Pearcey process again through the introduction of integrable deformations of the measure.

The second part of this monograph is mainly concerned with the spectral theory of random matrices, but ideas and methods from the theory of integrable systems plays a prominent role. The introductory chapter, by Harold Widom, begins with a review of basic operator theory definitions and results that are required for applications to random matrices. Then derivations are given for spacing distributions between consecutive eigenvalues, in term of gap probabilities. Using operatorial methods, these are expressed as Fredholm determinants, in suitable scaling limits, of integral operators with integrable kernels of sine type (for the bulk) and Airy type (leading to the Tracy-Widom distributions) for the edge. All three cases, orthogonal ($\beta = 1$), unitary ($\beta = 2$) and symplectic ($\beta = 4$) ensembles are treated. Finally, differential equations for distribution functions are derived, in particular, equations of Painlevé type.

In his series of chapters, Pavel Bleher gives a detailed survey of the use of Riemann–Hilbert methods for the study of the asymptotics of spectral distributions of random matrices. First, unitary ensembles with polynomial potentials are treated, and their relation to orthogonal polynomials and the associated Christoffel–Darboux kernels determining the correlation functions at finite N , as well as the string equations determining the recurrence coefficients in the asymptotic $1/N$ series. The Riemann–Hilbert characterization of the orthogonal polynomials is then introduced, and it is shown that the equilibrium measure is supported on a finite union of intervals coinciding with the cuts defining a hyperelliptic algebraic curve. In particular, the Wigner semi-circle law is derived for the Gaussian case, and the case of quartic potentials is treated in detail. There follows the treatment of scaled large N asymptotics of orthogonal polynomials, using the Riemann–Hilbert approach and the method of nonlinear steepest descent of Deift, Kriecherbauer, McLaughlin, Venakides Zhou (DKMVZ). A solution of the model Riemann–Hilbert problem is given in terms of Riemann theta functions. The Airy parametrix at the end points is constructed to complete the study of uniform asymptotics, and an indication of the proof of sine kernel universality in the bulk and Airy kernel universality at the edge.

Next, the double scaling limit for the critical point of the even quartic potential case is treated, and its relation to the Hastings–McLeod solution of the P_{II} Painlevé equation derived. More generally, the asymptotics of the free energy in the one cut case is studied (assuming a special regularity property of the potential). Analyticity in a parameter defining the potential is examined for the q -cut case for the density function and the free energy. The quartic deviation in the free energy from the Gaussian case is expressed in terms of the first two terms of the large N asymptotic expansion, and related to the Tracy–Widom distribution with an error estimate.

Random matrix models with exponential external coupling are then analyzed in terms of multiple orthogonal polynomials, with emphasis on the case of two distinct eigenvalues in the externally coupled matrix. Correlation functions, at finite N , are expressed in determinantal form in terms of an analog of the Christoffel–Darboux kernel. The higher rank Riemann–Hilbert

characterization of such multiple orthogonal polynomials is given, and the differential equations and recursion relations for these expressed in terms of these matrices. The relation to Brownian bridges is explained, and, finally, the Pearcey kernel is derived in the double scaling limit using the nonlinear steepest descent method.

Alexander Its, in his series, focuses upon the large N asymptotics of the spectra of random matrices. The reduced N -fold integral representation of the partition function of Hermitian matrix models is recalled, and the expression of eigenvalue correlation functions in terms of the Christoffel–Darboux kernel of the associated orthogonal polynomials. An introduction to the Its–Kitaev–Fokas Riemann–Hilbert approach to orthogonal polynomials is given, and a proof is given of its unique solvability under certain assumptions.

The asymptotic analysis, along the lines of the DKMVZ method, is then recalled, based on the introduction of the g -function (essentially, the log-Coulomb energy of the equilibrium distribution) to transform the exact RH problem into one which, in leading order, has only jump discontinuities, and hence may be solved exactly. A detailed analysis is then given for the case of even quartic potentials. This suffices to deduce the sine kernel asymptotic form for the correlation kernel in the bulk. The construction of the Airy parametrix at the end points of the cuts is then discussed and an asymptotic solution given with uniform estimates in each region.

Bertrand Eynard reviews the relationship between convergent matrix integrals and formal matrix integrals, serving as generating functions for the combinatorics of maps. Essentially, the formal model is obtained by treating the integrand as a perturbation series about the Gaussian measure and interchanging the orders of integration and summation, without regard to convergence. He indicates the derivation of the loop equations relating the expectation values of invariant polynomials of various degrees as Dyson–Schwinger equations. This is illustrated by various examples, including 1 and 2 matrix models, as well as chains of matrices, the $O(N)$ chain model and certain statistical models such as the Potts model. He ends with a number of interesting conjectures about the solution of the loop equations. In the current literature, this has led to a very remarkable program in which many results on random matrices, solvable statistical models, combinatorial, topological and representation theoretical generating functions may be included as part of a general scheme, based on the properties of Riemann surfaces, and their deformations.

The contribution of Momar Dieng and Craig Tracy deals in part with the earliest appearance of random matrices, due to Wishart, in the theory of multivariate statistics, the so-called Wishart distribution. They present Johnstone’s result relating the largest component variance to the F_1 Tracy–Widom distribution, as well as Soshnikov’s generalization to lower components. The expression of the F_1 , F_2 and F_4 distributions for the edge distributions in GOE, GUE and GSE respectively in terms of the Hastings–McLeod solution of the P_{II} Painlevé equation is recalled. There follow a discussion of

the recurrence relations of Dieng which enter in the computation of the m th largest eigenvalues in GOE and GSE.

A derivation of the Airy kernel for the edge scaling limit of GUE from Plancherel–Rotach asymptotics of the Hermite polynomials is given, as well as the P_I equation that determines the Fredholm determinant of the Airy kernel integral operator supported on a semi-infinite interval. The computation of the m th largest eigenvalue distribution in the GSE and GOE is indicated, together with an interlacing property identifying the first sequence with the even terms of the second. Finally, numerical results are given comparing these distributions with empirical data.

This volume is a masterly combined effort by several of the leading contributors to this remarkable domain, covering a range of topics and applications that no individual author could hope to encompass. For the reader wishing to have a representative view of the fascinating ongoing developments in this domain, as well as a reliable account of the, many results that are by now classically established, it should provide an excellent reference and entry point to the subject.

References

1. J. Harnad and M. Bertola (eds.) *Special Issue on Random Matrices and Integrable Systems* J. Phys. A **39** (2006).

Contents

Preface

<i>John Harnad</i>	V
References	IX

Part I Random Matrices, Random Processes and Integrable Models

1 Random and Integrable Models in Mathematics and Physics

<i>Pierre van Moerbeke</i>	3
1.1 Permutations, Words, Generalized Permutations and Percolation ...	4
1.1.1 Longest Increasing Subsequences in Permutations, Words and Generalized Permutations	4
1.1.2 Young Diagrams and Schur Polynomials	6
1.1.3 Robinson–Schensted–Knuth Correspondence for Generalized Permutations	9
1.1.4 The Cauchy Identity	11
1.1.5 Uniform Probability on Permutations, Plancherel Measure and Random Walks	13
1.1.6 Probability Measure on Words	22
1.1.7 Generalized Permutations, Percolation and Growth Models ...	24
1.2 Probability on Partitions, Toeplitz and Fredholm Determinants	33
1.2.1 Probability on Partitions Expressed as Toeplitz Determinants	35
1.2.2 The Calculus of Infinite Wedge Spaces	39
1.2.3 Probability on Partitions Expressed as Fredholm Determinants	44
1.2.4 Probability on Partitions Expressed as $U(n)$ Integrals	48

1.3 Examples 50

 1.3.1 Plancherel Measure and Gessel’s Theorem 50

 1.3.2 Probability on Random Words 54

 1.3.3 Percolation 56

1.4 Limit Theorems 60

 1.4.1 Limit for Plancherel Measure 60

 1.4.2 Limit Theorem for Longest Increasing Sequences 64

 1.4.3 Limit Theorem for the Geometrically Distributed Percolation Model, when One Side of the Matrix Tends to ∞ 67

 1.4.4 Limit Theorem for the Geometrically Distributed Percolation Model, when Both Sides of the Matrix Tend to ∞ 71

 1.4.5 Limit Theorem for the Exponentially Distributed Percolation Model, when Both Sides of the Matrix tend to ∞ 75

1.5 Orthogonal Polynomials for a Time-Dependent Weight and the KP Equation 76

 1.5.1 Orthogonal Polynomials 76

 1.5.2 Time-Dependent Orthogonal Polynomials and the KP Equation 81

1.6 Virasoro Constraints 88

 1.6.1 Virasoro Constraints for β -Integrals 88

 1.6.2 Examples 93

1.7 Random Matrices 96

 1.7.1 Haar Measure on the Space \mathcal{H}_n of Hermitian Matrices 96

 1.7.2 Random Hermitian Ensemble 99

 1.7.3 Reproducing Kernels 102

 1.7.4 Correlations and Fredholm Determinants 104

1.8 The Distribution of Hermitian Matrix Ensembles 108

 1.8.1 Classical Hermitian Matrix Ensembles 108

 1.8.2 The Probability for the Classical Hermitian Random Ensembles and PDEs Generalizing Painlevé 113

 1.8.3 Chazy and Painlevé Equations 119

1.9 Large Hermitian Matrix Ensembles 120

 1.9.1 Equilibrium Measure for GUE and Wigner’s Semi-Circle 120

 1.9.2 Soft Edge Scaling Limit for GUE and the Tracy–Widom Distribution 122

References 128

2 Integrable Systems, Random Matrices, and Random Processes

Mark Adler 131

2.1 Matrix Integrals and Solitons 134

 2.1.1 Random Matrix Ensembles 134

 2.1.2 Large n -limits 137

 2.1.3 KP Hierarchy 139

2.1.4 Vertex Operators, Soliton Formulas and Fredholm Determinants 141

2.1.5 Virasoro Relations Satisfied by the Fredholm Determinant ... 144

2.1.6 Differential Equations for the Probability in Scaling Limits ... 146

2.2 Recursion Relations for Unitary Integrals 151

2.2.1 Results Concerning Unitary Integrals 151

2.2.2 Examples from Combinatorics 154

2.2.3 Bi-orthogonal Polynomials on the Circle and the Toeplitz Lattice 157

2.2.4 Virasoro Constraints and Difference Relations 159

2.2.5 Singularity Confinement of Recursion Relations 163

2.3 Coupled Random Matrices and the 2-Toda Lattice 167

2.3.1 Main Results for Coupled Random Matrices 167

2.3.2 Link with the 2-Toda Hierarchy 168

2.3.3 $L-U$ Decomposition of the Moment Matrix, Bi-orthogonal Polynomials and 2-Toda Wave Operators 171

2.3.4 Bilinear Identities and τ -function PDEs 174

2.3.5 Virasoro Constraints for the τ -functions 176

2.3.6 Consequences of the Virasoro Relations 179

2.3.7 Final Equations 181

2.4 Dyson Brownian Motion and the Airy Process 182

2.4.1 Processes 182

2.4.2 PDEs and Asymptotics for the Processes 189

2.4.3 Proof of the Results 192

2.5 The Pearcey Distribution 199

2.5.1 GUE with an External Source and Brownian Motion 199

2.5.2 MOPS and a Riemann–Hilbert Problem 202

2.5.3 Results Concerning Universal Behavior 204

2.5.4 3-KP Deformation of the Random Matrix Problem 208

2.5.5 Virasoro Constraints for the Integrable Deformations 213

2.5.6 A PDE for the Gaussian Ensemble with External Source and the Pearcey PDE 218

A Hirota Symbol Residue Identity 221

References 223

Part II Random Matrices and Applications

3 Integral Operators in Random Matrix Theory

Harold Widom 229

3.1 Hilbert–Schmidt and Trace Class Operators. Trace and Determinant. Fredholm Determinants of Integral Operators 229

3.2 Correlation Functions and Kernels of Integral Operators. Spacing Distributions as Operator Determinants. The Sine and Airy Kernels 238

3.3 Differential Equations for Distribution Functions Arising in Random Matrix Theory. Representations in Terms of Painlevé Functions 243

References 249

4 Lectures on Random Matrix Models

Pavel M. Bleher 251

4.1 Random Matrix Models and Orthogonal Polynomials 252

 4.1.1 Unitary Ensembles of Random Matrices 252

 4.1.2 The Riemann–Hilbert Problem for Orthogonal Polynomials 260

 4.1.3 Distribution of Eigenvalues and Equilibrium Measure 263

4.2 Large N Asymptotics of Orthogonal Polynomials. The Riemann–Hilbert Approach 267

 4.2.1 Heine’s Formula for Orthogonal Polynomials 267

 4.2.2 First Transformation of the RH Problem 269

 4.2.3 Second Transformation of the RHP: Opening of Lenses 271

 4.2.4 Model RHP 272

 4.2.5 Construction of a Parametrix at Edge Points 280

 4.2.6 Third and Final Transformation of the RHP 286

 4.2.7 Solution of the RHP for $R_N(z)$ 287

 4.2.8 Asymptotics of the Recurrent Coefficients 288

 4.2.9 Universality in the Random Matrix Model 291

4.3 Double Scaling Limit in a Random Matrix Model 294

 4.3.1 Ansatz of the Double Scaling Limit 294

 4.3.2 Construction of the Parametrix in Ω^{WKB} 297

 4.3.3 Construction of the Parametrix near the Turning Points 299

 4.3.4 Construction of the Parametrix near the Critical Point 300

4.4 Large N Asymptotics of the Partition Function of Random Matrix Models 308

 4.4.1 Partition Function 308

 4.4.2 Analyticity of the Free Energy for Regular V 310

 4.4.3 Topological Expansion 311

 4.4.4 One-Sided Analyticity at a Critical Point 313

 4.4.5 Double Scaling Limit of the Free Energy 315

4.5 Random Matrix Model with External Source 315

 4.5.1 Random Matrix Model with External Source and Multiple Orthogonal Polynomials 315

 4.5.2 Gaussian Matrix Model with External Source and Non-Intersecting Brownian Bridges 321

 4.5.3 Gaussian Model with External Source. Main Results 322

 4.5.4 Construction of a Parametrix in the Case $a > 1$ 326

4.5.5 Construction of a Parametrix in the Case $a < 1$ 333
 4.5.6 Double Scaling Limit at $a = 1$ 340
 4.5.7 Concluding Remarks 346
 References 347

5 Large N Asymptotics in Random Matrices

Alexander R. Its 351
 5.1 The RH Representation of the Orthogonal Polynomials
 and Matrix Models 351
 5.1.1 Introduction 351
 5.1.2 The RH Representation of the Orthogonal Polynomials 355
 5.1.3 Elements of the RH Theory 360
 5.2 The Asymptotic Analysis of the RH Problem.
 The DKMVZ Method 373
 5.2.1 A Naive Approach 373
 5.2.2 The g -Function 373
 5.2.3 Construction of the g -Function 378
 5.3 The Parametrix at the End Points. The Conclusion of the
 Asymptotic Analysis 383
 5.3.1 The Model Problem Near $z = z_0$ 383
 5.3.2 Solution of the Model Problem 386
 5.3.3 The Final Formula for the Parametrix 390
 5.3.4 The Conclusion of the Asymptotic Analysis 391
 5.4 The Critical Case. The Double Scaling Limit and the Second
 Painlevé Equation 394
 5.4.1 The Parametrix at $z = 0$ 394
 5.4.2 The Conclusion of the Asymptotic Analysis
 in the Critical Case 399
 5.4.3 Analysis of the RH Problem (1^c)–(3^c). The Second Painlevé
 Equation 403
 5.4.4 The Painlevé Asymptotics of the Recurrence Coefficients 406
 References 412

6 Formal Matrix Integrals and Combinatorics of Maps

B. Eynard 415
 6.1 Introduction 415
 6.2 Formal Matrix Integrals 417
 6.2.1 Combinatorics of Maps 419
 6.2.2 Topological Expansion 423
 6.3 Loop Equations 423
 6.4 Examples 429
 6.4.1 1-Matrix Model 429
 6.4.2 2-Matrix Model 431
 6.4.3 Chain of Matrices 434
 6.4.4 Closed Chain of Matrices 435

6.4.5 $O(n)$ Model 435

6.4.6 Potts Model 437

6.4.7 3-Color Model 438

6.4.8 6-Vertex Model 438

6.4.9 ADE Models 438

6.4.10 ABAB Models 439

6.5 Discussion 439

6.5.1 Summary of Some Known Results 439

6.5.2 Some Open Problems 440

References 441

7 Application of Random Matrix Theory to Multivariate Statistics

Momar Dieng and Craig A. Tracy 443

7.1 Multivariate Statistics 443

7.1.1 Wishart Distribution 443

7.1.2 An Example with $\Sigma \neq cI_p$ 446

7.2 Edge Distribution Functions 448

7.2.1 Summary of Fredholm Determinant Representations 448

7.2.2 Universality Theorems 449

7.3 Painlevé Representations: A Summary 451

7.4 Preliminaries 454

7.4.1 Determinant Matters 454

7.4.2 Recursion Formula for the Eigenvalue Distributions 455

7.5 The Distribution of the m th Largest Eigenvalue in the GUE 458

7.5.1 The Distribution Function as a Fredholm Determinant 458

7.5.2 Edge Scaling and Differential Equations 459

7.6 The Distribution of the m th Largest Eigenvalue in the GSE 463

7.6.1 The Distribution Function as a Fredholm Determinant 463

7.6.2 Gaussian Specialization 468

7.6.3 Edge Scaling 474

7.7 The Distribution of the m th Largest Eigenvalue in the GOE 481

7.7.1 The Distribution Function as a Fredholm Determinant 481

7.7.2 Gaussian Specialization 486

7.7.3 Edge Scaling 490

7.8 An Interlacing Property 499

7.9 Numerics 503

7.9.1 Partial Derivatives of $q(x, \lambda)$ 503

7.9.2 Algorithms 503

7.9.3 Tables 504

References 505

Index 509

List of Contributors

Mark Adler

Department of Mathematics
Brandeis University
Waltham, MA 02454
USA
adler@brandeis.edu

Pavel M. Bleher

Department of Mathematical
Sciences
Indiana University–Purdue
University Indianapolis
402 N. Blackford St.
Indianapolis, IN 46202-3216
USA
bleher@math.iupui.edu

Momar Dieng

Department of Mathematics
The University of Arizona
617 N. Santa Rita Ave.
Tucson, AZ 85721-0089
USA
momar@math.arizona.edu

B. Eynard

Service de Physique Théorique
CEA/DSM/SPhT, CEA/Saclay
91191 Gif-sur-Yvette Cedex
France
eynard@spht.saclay.cea.fr

John Harnad

Department of Mathematics and
Statistics
Concordia University
1455 de Maisonneuve Blvd. West
Montréal, Québec, H3G 1M8
Canada
and
Centre de Recherches Mathématiques
Université de Montréal
C.P. 6128, Succ. Centre ville
Montréal, Québec, H3C 3J7
Canada
harnad@crm.umontreal.ca

Alexander R. Its

Department of Mathematics Sciences
Indiana University–Purdue
University Indianapolis
402 N. Blackford St.
Indianapolis, IN 46202-3216
USA
itsa@math.iupui.edu

Craig A. Tracy

Department of Mathematics
University of California
One Shields Ave.
Davis, CA 95616
USA
tracy@math.ucdavis.edu

Pierre van Moerbeke

Département de Mathématiques
Université Catholique de Louvain
1348 Louvain-la-Neuve
Belgium
vanmoerbeke@math.ucl.ac.be

Harold Widom

Mathematics Department
194 Baskin Engineering
University of California
Santa Cruz, CA 95064
USA
widom@math.ucsc.edu