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Volume 258

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Computational Homogenization of Heterogeneous Materials with Finite Elements

Springer

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To my wife Cécile and to my sons, Quentin, Nathan and Lucas.

Foreword

Multiscale mechanics as a discipline has grown tremendously in the past decades. With the steady increase of computational resources, it is nowadays possible to carry out mechanical analyses at multiple scales, whereby fine-scale details of the heterogeneous materials automatically impact the engineering response of a structure. Many journal publications emerged over the past years, which are not always immediately accessible to a larger audience and, in particular, not to fresh students who want to get acquainted with the subject. This justifies the need for a book like this one, where elementary aspects of computational homogenization and its various applications are given full attention.

This monograph from Julien Yvonnet focuses on a particular class of multiscale methods, primarily targeting the multiscale identification of various material properties. Computational homogenization is here defined in the broad sense, i.e., where macroscopic properties are extracted from microstructures using computational tools. The tools used here are based on the most widespread methodology, i.e., finite element based.

The topics covered in the book are rich. Starting from an introduction on some historical notes, the context of the book is clearly defined along with its application field and associated software tools. For the non-FEM experts, the second chapter reviews some elementary concepts of the finite element method, which is used throughout the book. The main emphasis is put on linear problems, which are widely used in engineering applications. The problems addressed include elasticity, thermoelasticity, thermal conductivity, piezoelectricity, porous media, and linear viscoelastic materials. The last two chapters touch on more advanced topics, i.e., the absence of scale separation (entailing a parallel calculation approach) and the two-scale nonlinear computational homogenization method. The nested solution at two scales, also known as FE^2, is detailed here as well. Finally, the nonlinear transformation field analysis method and alternative data-driven approaches are presented. All these topics have been addressed in 250 pages, leading to a book that becomes appealing for students, faculty, and industrial practitioners. Full attention is given to numerical details, which are not always reported in the literature. This allows for an easy implementation in simple codes.

There are not many books on computational homogenization that start from the basics and conclude with the state of the art. I think the author succeeded to put all most important theory, assumptions, solution methods, and computational details in this concise monograph. It is a fully self-contained book that can be used for education and exploitation. The book is, therefore, unique in many ways. I am convinced that it will serve as an educational guideline for some and an inspirational source for others.

Eindhoven, The Netherlands
January 2019

Marc Geers
Eindhoven University of Technology

Preface

In recent years, a revolution has occurred in the field of material engineering. This breakthrough is the possibility to design, predict properties, and even manufacture materials not only with the help of experiments but also with computer science. Until recently, determining mechanical properties of a material like stiffness or strength was only accessible through mechanical testing. In that context, selection or manufacturing of new materials with enhanced properties was a long trial–error process, requiring deep experience and intuition about elementary phenomena and influence of constituents. Due to the conjunction of several advances in physical/mechanical modeling, computational capabilities, numerical/mathematical methods, and the development of new manufacturing processes like 3D printing equipment, it is now possible to predict with good accuracy the properties of a large range of man-made materials like polymer/ceramic/metallic composites, concrete, or nanostructured materials and manufacture them with controlled microstructures.

While analytical micromechanical models have successfully helped to predict effective properties of heterogeneous materials for idealized microstructures, computational methods, and computer capacities permit to go beyond restricting assumptions and to consider realistic or architectured microstructures, with complex behaviors like elastoplasticity, damage, microcracking, or phase change. Another new possibility is to take into account several space or timescales, or several models like atomistic and continuum.

However, while these developments are quite mature in the research community, their transfer to industry is taking its first faltering steps. One reason is that many difficulties still remain for reaching solutions to central problems to industrial questions, like predicting damage from microcracking, prediction of fatigue, and taking into account the complexity of microstructures with all their uncertainties in materials like concrete. Another factor is that simulations related to complex material microstructures still involve extensive computational resources in terms of memory and times, and cannot yet be incorporated in fast user codes.

Roughly speaking, computational homogenization, as described in more detail in Chap. 1, precisely aims at evaluating the macroscopic properties (at the scale of engineering products), from the knowledge of the material composition at the “micro” scale, i.e., at the scale of constituents.

This monograph aims at providing a concise overview of the main theoretical and numerical tools to solve homogenization problems in solids with the finite element method, which is nowadays one of the most popular and used simulation methods in computational engineering sciences. Starting from simple cases (linear thermal case), the problems progressively become more complex to finish with nonlinear problems. The first part of the book (Chaps. 2–6) is not a compilation of current research in that field, but is intended to be a course, summarizing established knowledge in this area such that students at the graduate level, researchers, or engineers, who would like to start working on this subject will acquire the basics without any preliminary knowledge neither about homogenization nor finite elements. More specifically, the book is written with the objective of practical implementation of the methodologies in simple programs such as e.g., MATLAB®. Then, the presentation is kept at a level where no deep mathematics is required. The second part of the book (Chaps. 7–9) is dedicated to more recent methodologies.

The main features of this book are as follows:

- Presentation of the theories and methodologies starting from simple linear cases up to more complex cases (multiphysics, nonlinear), including many points not found in other books.
- The book is self-contained and all details to practically implement the numerical algorithms are provided.
- Reference solutions are provided at the end of most chapters such that the user can validate his own code.

This book has been mainly written on the basis of two courses I gave at the graduate level at Université Paris-Est since 2011. Another part of the book is based on my group’s research works.

Champs-sur-Marne, France Julien Yvonnet
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