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# Fractal Apertures in Waveguides, Conducting Screens and Cavities

Analysis and Design

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# Preface

Fractal geometries have widely been used in electromagnetics, specifically, in antennas and frequency selective surfaces (FSS). The self-similarity of fractal geometry gives rise to a multiband response, whereas the space-filling nature of the fractal geometries makes it an efficient element in antenna and FSS unit cell miniaturization. Till date, no effort has been made to study the behavior of these fractal geometries in the aperture coupling problems. Aperture coupling problem is an important boundary value problem in electromagnetics, and has widely been used in waveguide filters and power dividers, slotted ground planes, frequency selective surfaces, and metamaterials. The present work is primarily intended to initiate a study on the characteristics of fractal apertures in waveguides, conducting screens, and cavities.

In order to carry out a unified analysis of these entirely dissimilar problems, the “generalized network formulation for the aperture problems” proposed by Mautz and Harrington has been extended to multiple-aperture geometry. We have considered the problem of coupling between two arbitrary regions coupled together via multiple apertures of arbitrary shape. The two regions are decoupled by the application of equivalence principle and enforcement of boundary conditions over the aperture regions that lead to an operator equation. The operator equation is then reduced to matrix form via the Method of Moments (MoM). The general problem of coupling through apertures is formulated in such a manner that only part of the problem needs to be reformulated for the solution of different problems like fractal diaphragms in a rectangular waveguide, radiation from waveguide-fed fractal apertures, coupling through fractal apertures in a conducting screen, and radiation from cavity-backed fractal apertures. A Galerkin procedure with rooftop and Rao-Wilton-Glisson (RWG) functions has been used for the first three problems, while a hybrid method using Finite Element Method (FEM) and MoM has been used for the cavity-backed fractal aperture problem. MATLAB codes have been developed for the problems and validated with the results available in the literature as well as through simulation on ANSOFT’s HFSS.

The use of resonant apertures in the transverse cross-section of rectangular waveguide improves the out-of-band rejection ratio of waveguide filters and also results in more compact and lightweight waveguide filters. The multiple aperture irises further improve the out-of-band rejection due to the formation of rejection band. Till now, some regular geometries have been analyzed in the literature. Due

to requirement of multiple aperture iris and compact waveguide filters, we have studied the behavior of fractal apertures in the transverse cross-section of a waveguide. Two types of basis functions have been used in the analysis. In the first case, the rooftop functions are used to model rectangular aperture problem in which the aperture surface is discretized in small rectangular sub-areas. The integrals involving the Green's function are calculated analytically over the rectangular domain. In the second approach, the apertures are discretized in triangular subareas in order to model the arbitrary aperture surface and RWG functions are used as the basis functions. The integrals over triangular domain are calculated numerically using Gaussian quadrature. The frequency responses of several fractal apertures are studied and some novel self-affine fractal geometries are proposed to obtain multiband response. A study has also been carried out to find a relation between the scale factor of the fractal geometry and the frequency response of the waveguide diaphragm. The study has been further extended to analyze the performance of finite periodic arrangement of these fractal apertures in the design of multiband waveguide filters and electromagnetic bandgap structures (EBG).

The problem of electromagnetic coupling between two regions via apertures in infinite screen has widely been used in the design of FSS, antenna arrays, and slotted ground plane. An infinite screen perforated with multiple apertures has a bandpass response when illuminated by a plane wave of varying frequency. The coupling through fractal apertures in an infinite screen has been solved with RWG basis functions. In this case, the integrals involving the free space Green's function suffer from singularity problem. Singularity cancellation method has been used to calculate the integral, which has the advantage of being accurate and, also, the calculation of these integrals over triangular domain can be done with a purely numerical technique. Several self-similar and space-filling fractal geometries are studied and, also, the effect of the variation of angle of incidence on the frequency response of fractal apertures has been investigated. It has been found that the variation of angle of incidence affects the performance of fractal apertures and some additional pass bands arise for inclined incidence. It is found that the fractal apertures support subwavelength transmission of electromagnetic waves and this property is more prominent in the space-filling fractal apertures. The coupling through the fractal apertures in infinite screen has been extended to the case of radiation through fractal apertures in infinite screen. It has been found that radiation from such fractal apertures improves the antenna input matching and dual-band waveguide radiators can easily be realized.

Cavity-backed aperture antennas are very popular in aerospace applications due to their conformal nature. The cavity-backed aperture antenna satisfies the requirements of being flush mounted as well as lightweight and small size. Also, the use of metallic cavity makes the radiation pattern unidirectional. Another most important advantage of the cavity-backed apertures is that they offer very small mutual coupling between the elements and are useful in the design of phased arrays. Cavity-backed fractal aperture is another field, which could be explored to design small-sized multiband antennas. The problem has been formulated using combined FEM and MoM method. In the combined FEM/MoM method, the

electric field inside the cavity is found using FEM and the surface magnetic current over the aperture surface is calculated using MoM. For FEM formulation, the cavity has been discretized into tetrahedral elements and the apertures into triangular elements. The simultaneous equations obtained over the subdomains are added to form the global matrix equation. This procedure gives a partly sparse and a partly dense matrix, which is then solved to find the unknown electric field over the apertures. The radiation characteristics and input characteristics of the antenna are then calculated from the electric field. The performance of cavity-backed fractal apertures with a coaxial probe feeding has been investigated. The numerical results are again validated with simulation results on HFSS. A novel effort has been made to relate the electromagnetic behavior of the fractal aperture with the scale factor of the geometry. It has been found that the location of resonant frequency of the antenna can be changed by changing the scale factor of fractal apertures.

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