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Reflection Positivity

A Representation Theoretic Perspective

Springer

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Preface

The concept of reflection positivity (RP) occurs as an important theme in various areas of mathematics and physics:

- In the representation theory of Lie groups, it establishes a passage of a unitary representation of a symmetric Lie group (such as the euclidean motion group) to a unitary representation of the Cartan dual group (such as the Poincaré group) [FOS83, JOl00, JOl98, NÓ14, JPT15].
- In constructive Quantum Field Theorem (QFT), it arises as the condition of Osterwalder–Schrader (OS) positivity for a euclidean field theory to correspond to a relativistic one [GJ81, Ja08, Ja18, Os95a, Os95b, OS73, OS75].
- For stochastic processes, it is weaker than the Markov property and specifies processes arising in lattice gauge theory. It plays a central role in the mathematical study of phase transitions and symmetry breaking [FILS78, JJ16, JJ17, Nel73].
- In analysis, it is a crucial condition that leads to inequalities such as the Hardy–Littlewood–Sobolev inequality [FL10].

Only recently, it became apparent that there are many hidden and still not sufficiently well-understood structures underlying the duality between unitary representations of a symmetric Lie group and its dual. Establishing reflection positivity in this context requires new analytic methods and new geometric insight into constructions and realizations of representations in analytic contexts. New developments concern analytic issues such as criteria for integrating Lie algebra representations to Lie group representations, reflection positive functions, distributions and kernels, new dilation techniques for representations and unexpected connections between Kubo–Martin–Schwinger (KMS) states of C^*-algebras and reflection positive unitary representations.

This was our motivation to write this “light” introduction to the representation theoretic aspects of reflection positivity to present this perspective on a level suitable for doctoral students.

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