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# Complex Scheduling

With 135 Figures  
and 3 Tables

 Springer

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# Preface

Scheduling problems have been investigated since the late fifties. Two types of applications have mainly motivated research in this area: project planning and machine scheduling. While in machine scheduling a large number of specific scheduling situations depending on the machine environment and the job characteristics have been considered, the early work in project planning investigated scheduling situations with precedence constraints between activities assuming that sufficient resources are available to perform the activities. More recently, in project scheduling scarce resources have been taken into account leading to so-called resource-constrained project scheduling problems. On the other hand, also in machine scheduling more general and complex problems have been investigated. Due to these developments today both areas are much closer to each other. Furthermore, applications like timetabling, rostering or industrial scheduling are connected to both areas.

This book deals with such complex scheduling problems and methods to solve them. It consists of three parts: The first part (Chapters 1 and 2) contains a description of basic scheduling models with applications and an introduction into discrete optimization (covering complexity, shortest path algorithms, linear programming, network flow algorithms and general optimization methods). In the second part (Chapter 3) resource-constrained project scheduling problems are considered. Especially, methods like constraint propagation, branch-and-bound algorithms and heuristic procedures are described. Furthermore, lower bounds and general objective functions are discussed. In the last part (Chapter 4) generalizations of the job-shop problem are covered leading to applications like job-shop problems with flexible machines, transport robots or with limited buffers. Heuristic methods to solve such complex scheduling problems are presented.

We are indebted to many people who have helped in preparing this book. Students in our courses during the last two years at the University of Osnabrück have given suggestions for improving earlier versions of this material. Andreas Drexl and Silvia Heitmann read carefully parts of the book and gave constructive comments.

We are grateful to the Deutsche Forschungsgemeinschaft for supporting the research that underlies much of this book. In addition we like to acknowledge the advice and support of Philippe Baptiste, Rainer Kolisch, Rolf H. Möhring, Klaus Neumann and Erwin Pesch.

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