

Lecture Notes in Physics

Edited by J. Ehlers, München, K. Hepp, Zürich
R. Kippenhahn, München, H. A. Weidenmüller, Heidelberg
and J. Zittartz, Köln
Managing Editor: W. Beiglböck, Heidelberg

79

Group Theoretical Methods in Physics

Sixth International Colloquium
Tübingen 1977

Edited by P. Kramer and A. Rieckers



Springer-Verlag
Berlin Heidelberg New York 1978

Editors

Prof. P. Kramer

Dr. A. Rieckers

Institut für Theoretische Physik

der Universität Tübingen

Auf der Morgenstelle 14

D-7400 Tübingen

Library of Congress Cataloging in Publication Data

International Colloquium on Group Theoretical
Methods in Physics, 6th, Tübingen, Ger., 1977.
Group theoretical methods in physics.

(Lecture notes in physics ; 79)

Bibliography: p.

Includes index.

1. Groups, Theory of--Congresses. 2. Symmetry
(Physics)--Congresses. I. Kramer, Peter,
1933- II. Rieckers, Alfred. III. Title.
IV. Series.
QC20.7.G76I57 1977 530.1'5'222 78-9742
ISBN 0-387-08848-2

ISBN 3-540-08848-2 Springer-Verlag Berlin Heidelberg New York

ISBN 0-387-08848-2 Springer-Verlag New York Heidelberg Berlin

This work is subject to copyright. All rights are reserved, whether the whole or part of the material is concerned, specifically those of translation, re-printing, re-use of illustrations, broadcasting, reproduction by photocopying machine or similar means, and storage in data banks. Under § 54 of the German Copyright Law where copies are made for other than private use, a fee is payable to the publisher, the amount of the fee to be determined by agreement with the publisher.

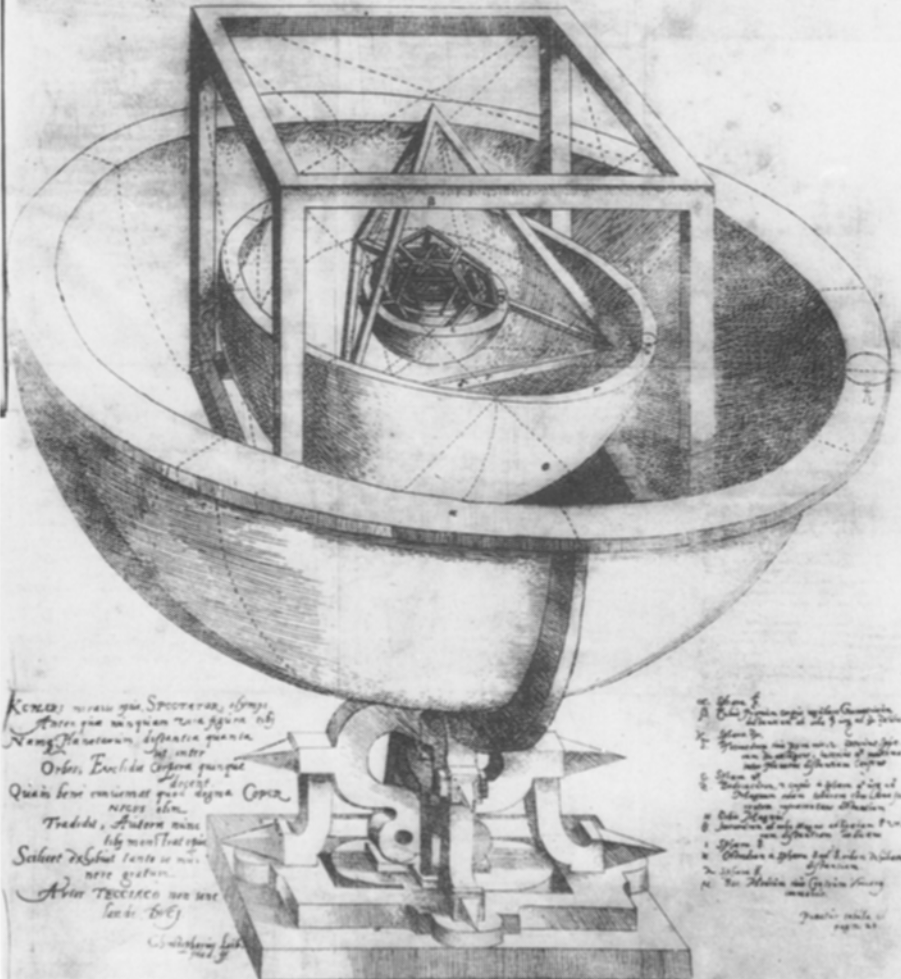
© by Springer-Verlag Berlin Heidelberg 1978

Printed in Germany

Printing and binding: Beltz Offsetdruck, Hemsbach/Bergstr.
2153/3140-543210

TABULA III. ORBIVM PLANETARVM DIMENSIONES, ET DISTANTIAS PER QVINQVE
REGVLARIA CORPORA GEOMETRICA EXHIBENS.

ILLVSTRISS: PRINCIPI, AC DÑO, DÑO FRIDERIGO, DVCI WIR-
TENBERGICO, ET TEGGIO, COMITI MONTIS BELGARVM, ETC. CONSECRATA.



*Kepler: miris qm Spectaret, et qm
Ante que unquam res agere ab
Nemo Planetarum, distantia quanta
est inter
Orbes, Euclida Capite quinquagesimo
Quam bene conueniet quae aequa Copernici
Nepos ibi.
Tradidit, Autorem vniuersi
libri mentis tractatu
Solut de Sphaera tanta se nō
nōre quatuor.
Aster Tractatu suo vniuersi
libri de Sphaera*

*1. Sphaera
2. Orbis
3. Planities
4. Sphaera
5. Sphaera
6. Sphaera
7. Sphaera
8. Sphaera
9. Sphaera
10. Sphaera
11. Sphaera
12. Sphaera*

Excudit Tubingae Georgius Griggenbachum M D C XVII

The colloquium calls back to mind Johannes Kepler's attempt to relate the distances between spheres of planets to the five platonic bodies. He noted that the cube, tetrahedron, dodecahedron, icosahedron and octahedron could be fitted in between the spheres of Copernicus. This coincidence he considered as a striking proof of the heliocentric hypothesis. Moreover he proposed to examine the kinematics assumed by Copernicus with respect to shortcomings responsible for deviations from the geometric pattern yielded by the platonic bodies. When Kepler, with the help of his teacher Michael Maestlin, published his construction in 1597 at Tübingen under the title "Mysterium Cosmographicum", he was already working at Graz. At that time he did not know that he would devote the whole of his life to the examination proposed, which led him to the study of planetary motion in general. The result was a new form of celestial kinematics with consequences perhaps more profound than those of the theory of Copernicus.

Although Kepler's present day fame rests on his laws of planetary motion, these laws give us only an inkling of the world-view from which they evolved. The directions of his work, as exemplified by his *Harmonice Mundi*, published in Linz in 1619 and containing the third law, were not well appreciated for a long time to come. He wanted to anchor his results on the ground of symmetries underlying the whole structure of the world. For symmetry he looked in regular and semiregular bodies, in tessellations of plane and space, and in the study of harmonies. He lived in a century when for the first time attention was paid to the geometrical shape of crystals. Facing the problem of hexagonal flakes of snow he tried to solve it by the hypothesis of closest packing of small particles.

The *Mysterium Cosmographicum* always ranged high in his own opinion. He failed in his hope that this work would open him the way back to Tübingen. After his great discoveries he published a revised edition in 1621 at Frankfurt. The *Colloquium* may show that his ideas are alive and give shape to wide areas of present-day physics.

M. Schramm, Professor of the History of Science, Tübingen

INTRODUCTION

The present volume comprises the proceedings of the VI. International Colloquium on Group Theoretical Methods in Physics held at Tübingen in July 1977 on the occasion of the five hundredth anniversary of the Eberhard-Karls-Universität. A glance at the table of contents shows the variety of contributions presented. It may be seen that many different branches of physics were dealt with. The methods of group theory which connect these contributions range from recent mathematical research to specific applications. The attempt at communication over a wide range of physical and mathematical concepts has been one of the characteristic features of this series of colloquia.

As in former sessions, specific perspectives to the present colloquium were given through the choice of a few main topics. These topics were selected by the advisory and organizing committee and represent actual developments in physics which are intrinsically connected with group theory. While each topic could serve as the subject of a conference, the intention of the colloquium was to communicate through coordinated plenary lectures to a broader audience the general mathematical and physical ideas involved as well as representative applications.

A short summary of the main topics and plenary lectures may serve for a first orientation:

A. Supersymmetry and graded Lie algebras

A survey on the structure of these algebras by V. Rittenberg is followed by a lecture on supergravity in particle physics and field theory by S. Ferrara. In a short contribution B. Kostant summarizes some remarks made at a discussion meeting on connections of harmonic analysis for supersymmetry with geometric quantization.

B. Concepts of order and disorder arising from molecular physics

The mathematical and physical aspects of a certain partial order relation were treated in lectures by A. Kerber and E. Ruch. For the physical applications we must refer to the references given by A. Kerber since the manuscript by E. Ruch has not been received. Two lectures are devoted to general kinematical aspects (H. Primas)

and non-rigid motion (H. Frei and H. Günthard) in molecular physics. Note that a generalized version of the mentioned partial order appears also in H. Primas' lecture. The connection between the lectures of H. Primas and G.G. Emch seems also worth mentioning.

C. Symplectic structures and many-body physics

Two lectures deal with the geometry of canonical transformations in quantum mechanics (A. Grossmann) and recent results on the metaplectic representation (S. Sternberg). Applications in nuclear physics deal with collective models (D.J. Rowe and G.J. Rosensteel) and composite particles (P. Kramer). The relation of S. Sternberg's lecture with supersymmetry should not be overlooked.

D. Symmetry breaking in statistical mechanics and field theory

A variety of conceptual approaches to this topic are treated in the lectures, including a review on continuous phase transitions (J. Birman), a topological classification of symmetry defects in crystals (L. Michel), the algebraic approach (G.G. Emch), and symmetry breaking in particle physics and field theory (H.P. Dürr).

E. Automata and systems as examples of applied (semi-)group theory

This field of research outside physics leans heavily on the analysis of semigroup structures and semigroup actions, and was introduced in order to stimulate the transfer of ideas with physics. The topics of the lectures are semigroups in the theory of automata (H. Jürgensen) and control semigroups acting on manifolds (C. Lobry).

F. Renormalization group

The lecture by J.M.J. van Leeuwen deals with position space renormalization for phase transitions. Unfortunately, K. Osterwalder was finally prevented to give a second lecture on the renormalization group in field theory.

G. Gauge theories

The lecture by L. O'Rai feartaigh on solitons concluded the sessions of the colloquium. An application of L. Michel's topological methods to the study of solitons was mentioned in the discussion. There are of course also many connections between topics A and G.

The realization of this coordinated program depended crucially on the cooperation of the speakers and their willingness to prepare widely understandable talks. The manuscripts of the first and larger part of this volume present the outcome of this attempt, and the reader is invited to form his own judgement on the structure of the program. If the intention of the colloquium has been achieved, the lectures contained in this volume could offer a counterbalance to the ever growing trend for specialization, a counterbalance that is lastly indispensable for the progress even in specialized research.

The second part of this volume contains the poster contributions to the colloquium. The poster sessions took an important part in the actual time schedule of the colloquium. It seems to have been an unanimous consent among the participants that this way of scientific interaction is fruitful and stimulating and should be recommended for future sessions. Since the coordination of the plenary program necessarily implies a restriction with respect to possible topics, the advisors and organizers agreed that no similar constraint should apply to the poster contributions. The contributions received are nevertheless centered around one of the following topics:

- I. Supersymmetry, symmetry in particle and relativistic physics
- II. Symmetry in molecular and solid state physics
- III. Broken symmetry and phase transitions
- IV. Structure of groups and dynamical systems
- V. Representations of groups and Lie algebras
- VI. General symmetries, quantization

The poster contributions appear in this volume in the form of three-page abstracts with the exception of two papers where the inclusion of tables seemed to be indispensable. This reduction in size was necessary to maintain the overall size of the proceedings but does not reflect the role played by the poster sessions for the success of the colloquium. It is believed that these abstracts form another essential part of the proceedings. The research work reported here, if published only in specific journals, would be almost inaccessible to the non-expert from another field.

Since the authors have been asked to include relevant references, these contributions could serve to stimulate the reader to get acquainted with recent work outside his specific area of research.

Finally, the editors would like to use this opportunity for a few words of thanks from the side of the organizing committee. The organizers enjoyed, from the first meeting at Tübingen in October 1976 to the session in July 1977, the continuous cooperation of the advisors. Their activity applied in particular to the formation of the program, to the proposal of plenary lectures, and to the invitations of speakers. The editors express their own and their colleagues full appreciation of this work. Partial financial support of the Deutsche Forschungsgemeinschaft for the scientific program is gratefully acknowledged. It is a pleasure to thank A. Theis, Präsident der Eberhard-Karls-Universität Tübingen, for financial support of the meeting, for providing the facilities of the university, and for giving the opening address of the VI. Colloquium. Thanks are also due to J. Lang for handling all the necessary relations between the organizers and the university. We were very glad to receive from Prof. M. Schramm the comments on Kepler's work. Last not least, we are indebted to Mrs. S. Leung for the excellent work done as the secretary of this colloquium.

These proceedings represent joint work of all the authors of plenary lectures and poster contributions, and the editors express their gratitude to all colleagues who contributed to this volume. For the publication of the proceedings in this series of Lecture Notes and for the preparation of the manuscripts we received much help from Prof. W. Beiglböck.

Tübingen, March 1978

The editors:

P. Kramer A. Rieckers

Advisory committee:

H. Bacry, Marseille
L.C. Biedenharn, Durham, N.C.
H.P. Dürr, München
A. Janner, Nijmegen
E. Ruch, Berlin
J. Wess, Karlsruhe

Organizing committee:

G. John
P. Kramer
A. Rieckers
K. Scheerer
H. Stumpf
M. Dal Cin

Colloquia on Group Theoretical Methods in Physics

	Location:	Proceedings:
I. 1972	Centre de Physique Théorique de CNRS in Marseille, France	Joint report of the University of Provence, the University of Aix-Marseille and the CNRS
II. 1973	University of Nijmegen, Nijmegen, Netherlands	Printed by the Faculty of Science, University of Nijmegen
III. 1974	Centre de Physique Théorique, Marseille, France	Printed by the Faculty of Science, University of Nijmegen
IV. 1975	University of Nijmegen, Nijmegen, Netherlands	Lecture Notes in Physics, No. 50 Springer 1976, ed. by A. Janner, T. Janssen, and M. Boon
V. 1976	Université de Montréal, Montréal, Canada	Academic Press 1977, ed. by R.T. Sharp and B. Kolman
VI. 1977	Universität Tübingen, Tübingen, West-Germany	This volume
VII. 1978	University of Texas, Austin, Texas, USA	

CONTENTS

Invited lectures

A. <u>Supersymmetry and graded Lie algebras</u>	1
1. V. Rittenberg: A guide to Lie superalgebras	3
2. S. Ferrara: Supergravity in the physics of particles and fields	22
3. B. Kostant: Harmonic analysis on graded (or super) Lie groups	47
B. <u>Concepts of symmetry and disorder arising from molecular physics</u>	51
1. A. Kerber: The diagram lattice as structural principle in mathematics	53
2. H. Primas: Kinematical symmetries in molecular quantum mechanics	72
3. H. Frei and Hs. H. Günthard: Symmetry of non-rigid molecules	92
C. <u>Symplectic structures and many-body physics</u>	115
1. S. Sternberg: Some recent results on the metaplectic representation	117
2. D.J. Rowe and G. Rosensteel: The nuclear collective model and the symplectic group	144
3. A. Grossmann: Geometry of real and complex canonical transformations in quantum mechanics	162
4. P. Kramer: Composite particles and symplectic (semi-)groups	180

D. <u>Symmetry breaking in statistical mechanics and field theory</u>	201
1. J.L. Birman: Group theory of the Landau-thermodynamic theory of continuous phase transitions in crystals	203
2. G.G. Emch: Phase transitions, approach to equilibrium, and structural stability	223
3. L. Michel: Topological classification of symmetry defects in ordered media	247
4. H.P. Dürr: Dynamical origin of symmetry	259
E. <u>Automata and systems as examples of applied (semi-)group theory</u>	283
1. C. Lobry: Action of control semigroups on manifold and application to realization theory	285
2. H. Jürgensen: Some applications of the theory of semigroups to automata	307
F. <u>Renormalization group and critical phenomena</u>	323
J.M.J. van Leeuwen: Position space renormalization group	325
G. <u>Gauge fields</u>	343
L. O'Raifeartaigh: Static solitons in more than one dimension	345

Contributed papers

I. <u>Supersymmetry, symmetry in particle and relativistic physics</u>	357
1. S. Albeverio and R. Høegh-Krohn: Remarks on the energy representation of Sobolev-Lie groups	359
2. J.P. Antoine and C. Malou: Unbounded representations of the Poincaré and Gauge groups in the indefinite metric quantization of the electro- magnetic field	361
3. N. Backhouse: On the construction of graded Lie algebras	364
4. L.C. Biedenharn and L.P. Horwitz: Exceptional parafermions in a Hilbert space over an associative algebra	367
5. G. Burdet and M. Perrin: The SU(4) nuclear symmetry revisited	370
6. J.P. Dahl: The spinning electron	373
7. M. Daumens, M. Perroud and P. Winternitz: Group theoretical expansions of scattering amplitudes for particles with spin	376
8. F. González-Gascón: On the extension of vector fields and the superluminal transformations	379
9. H. Hoogland: A group theoretical derivation of the minimal coupling in elementary quantum mechanics	382
10. H. Nicolai: Towards a constructive approach to supersymmetric ϕ^3	385
11. J. Patera, P. Winternitz and H. Zassenhaus: The maximal abelian subgroups of the conformal group of space-time	388
12. T.S. Santhanam: The exceptional groups as candidates for supersymmetry	391
13. Dj. Šijački: Quark bag excitations, SL(3,R) spectrum generating group and vector states in e^+e^- annihilation	394

II. <u>Symmetry in molecular and solid state physics</u>	397
1. M. Boon and J. Zak: Completeness of networks of states	399
2. P.M. van den Broek: PUA representations of Shubnikov space groups and selection rules	402
3. A.P. Cracknell and B.L. Davies: Computer programs for determining wave vector selection rules (WVSRs) for space groups	405
4. B.L. Davies and A.P. Cracknell: Results of computer programs for determining the reductions of the Kronecker products of the irreducible representations of space groups	408
5. R. Dirl: Compatibility relations for factor systems and space group representations	411
6. A. Janner and T. Janssen: Bravais lattices associated with incommensurate crystal phases	414
7. B.R. Judd: Ligand polarizations and lanthanide ion spectra	417
8. R.W.J. Roël: Basic exchange integrals and the triple double coset symbol	420
III. <u>Broken symmetry and phase transitions</u>	423
1. G. De Concini and G. Vitiello: Group Contractions and infrared effect in theories with spontaneous breakdown of symmetry	425
2. F. Constantinescu and H.M. Ruck: Instantons in lattice models with discrete symmetries	429
3. M. Hongoh and D. Matz: Spontaneous breakdown of symmetry and the generalized coherent states	432
4. M.V. Jarić and J.L. Birman: Molien function and calculation of invariant polynomials for space groups	436
5. A.J. Kálnay: Gauge fields and quantum liquids	441

6. P. Kasperkovitz: A new model of a structural phase transition	444
7. L. Michel and J. Mozrzymas: Application of Morse theory to the symmetry breaking in the Landau theory of second order phase transition	447
8. J.P. Provost and G. Vallee: Spontaneous breakdown of the gauge symmetry and observable phase operator	462
<u>IV. Structure of groups and dynamical systems</u>	465
1. M. Dal Cin and E. Dilger: Semigroups and effective structures of automata	467
2. L.L. Boyle and K.F. Green: Studies of some physically-relevant representation groups	470
3. E. Dilger: Effective decompositions of automata	473
4. M. Hazewinkel: On invariants and canonical forms for linear dynamical systems	476
5. B. Kümmerer: Mean ergodic semigroups of contractions in W^* -algebras	479
<u>V. Representations of groups and Lie algebras</u>	483
1. C. Bretin and J.P. Gazeau: About some series associated to complex semi-simple Lie algebras	485
2. J.P. Gazeau , M.Cl. Dumont-Lepage and A. Ronveaux: Gelfand lattice polynomials and finite irreducible representations of $GL(n, \mathbb{C})$	488
3. J.W.B. Hughes: Partitions of integers and Lie algebras	491
4. Y. Ilamed: Lie elements, the killing form and trace identities	494
5. R.C. King and A.H.A. Qubanchi: Branching Rule $SO(7) \rightarrow G_2$	497
6. W. Laskar: Simplified Racah's eigenvalue formula for the second order Casimir operator	500

7. K. Srinivasa Rao:	
A note on the series expansions for the Racah coefficient	503
8. J. Yadegar:	
Construction of $O(3)$ shift operators and their use in classification of Lie algebras	506
VI. <u>General symmetries, quantization</u>	509
1. F.J. Bloore and S. Swarbrick:	
Wave functions of identical particles	511
2. M. Brunet:	
The metaplectic Semigroup and the implementation of complex linear canonical transformations in quantum mechanics	512
3. G. Burdet and M. Perrin:	
Weyl quantization and metaplectic representation	515
4. U. Cattaneo:	
On quantum mechanical symmetry groups	518
5. P. Kramer, M. Moshinsky and T.H. Seligman:	
Non-bijective canonical transformations and their representations in quantum mechanics	521
6. U. Niederer:	
Kinematical symmetries of the nonlinear diffusion equations	522
7. J.F. Pommaret:	
Lie pseudogroups and the structure of physical laws	525
8. A. Rieckers:	
Equivalence of Kadison and Wigner symmetries in traditional quantum mechanics	528
9. M. Romerio:	
The coherent states associated with a compact semi-simple Lie group	531
10. R.N. Sen:	
Theory of symmetry of infinite systems	534
11. M.C. Singh:	
Group theoretic approach to similarity analysis with applications to the problems of wave propagation	537
12. I. Szczyrba:	
Generalized number operator in the Fock representation	540
13. J. Tolar:	
Quantization and deformation theory	543