

Lecture Notes in Physics

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6

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Group Representations in Mathematics and Physics

Battelle Seattle 1969 Rencontres
Edited by V. Bargmann, Princeton University



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PREFACE

The Battelle Seattle Summer Rencontres in Mathematics and Physics have been established in order "to provide a channel of communication between mathematicians and physicists". Each year a topic of common interest was chosen which--it was hoped--would lead to fruitful discussions.

The 1969 Rencontres dealt with "Group Representations in Mathematics and Physics". In the view of most participants, I believe, these Rencontres lived up to expectation. The discussions were lively, mathematicians and physicists found many questions of common interest to talk about, and they learned from each other. On the mathematical side the emphasis was on the problems of infinite-dimensional representations, a subject which has attracted and continues to attract many workers in functional analysis. On the side of physics the applications to quantum theory of both finite- and infinite-dimensional representations were considered, with particular attention to the theory of elementary particles, including its most recent branch, "current algebra", which leads beyond the framework of Lie groups and calls for new methods.

As it should be, individual contacts and informal gatherings played a major role during these Rencontres. In addition, there were four series of lectures--two in Mathematics (by E. M. Stein and by C. C. Moore) and two in Physics (by L. Michel and by L. O'Raifeartaigh)--and a number of seminar talks by various participants.

The present volume contains three of the four lecture series (see the Note below) and five seminar talks, written up by their authors. The reader is expected to be familiar with the main facts of the theory of group representations. On the other hand, to help the mathematician, Michel and O'Raifeartaigh have carefully

Note. Due to the pressure of other work Professor Stein was, unfortunately, unable to prepare his lecture notes for publication. His course of lectures was entitled "Special Topics in the Representation Theory of Semi-Simple Groups" and dealt primarily with the analytic continuation of representations. Professor Stein has kindly supplied the following list of his publications on this subject.

- (a) E. M. Stein, "Analytic Continuation of Group Representations", *Advances in Mathematics*, 4 (2), 172-207 (April, 1970). Academic Press, New York and London.
- (b) A. W. Knap and E. M. Stein, "Singular Integrals and the Principal Series I. and II.", I. *Proc. Nat. Acad. Sci. U.S.A.*, 63 (2), 281-284 (June, 1969); II. *Ibid.*, 66 (1), 13-17 (May, 1970).
- (c) A. W. Knap and E. M. Stein, "The Existence of Complementary Series". To appear in *Problems in Analysis*, Symposium in Honor of Solomon Bochner. Princeton University Press.

IV

stated the quantum theoretical postulates and described the experimental evidence on which the group theoretical treatment of quantum physics is based.

The contributions to this volume range from systematic accounts of extensive fields to reports on current research on very specific questions. Only the papers by Moore and by Kostant may be called "purely mathematical". All other papers--although highly mathematical in content--are concerned with, or at least motivated by physical problems. A few remarks on the interrelation between these more physical papers may help some readers. The papers by Michel and O'Raifeartaigh give a very comprehensive account of the application of group theory to the most varied problems of quantum physics. While the two papers are, in the main, complementary to each other they overlap in some measure, but wherever they do, the discussion of the questions they both treat is sufficiently different to be highly interesting and illuminating. The contributions by Goldin-Sharp and by Hermann are concerned with current algebra. An introduction to the concepts involved here may be found in the last chapter of Michel's paper and, in greater detail, in the last chapter of O'Raifeartaigh's paper. Lastly, the papers by Todorov and by Rühl are independent of each other and of the remaining papers in this volume.

On behalf of all participants I take great pleasure in thanking the officers and the staff of the Battelle Memorial Institute, who did so much to provide an ideal setting for these Rencontres.

Special thanks are due to the technical typing staff at the Battelle Seattle Research Center, for their painstaking efforts in preparing this volume for publication.

April, 1970

V. Bargmann

TABLE OF CONTENTS

CALVIN C. MOORE - Restrictions of Unitary Representations to Subgroups and Ergodic Theory: Group Extensions and Group Cohomology	1
LOUIS MICHEL - Applications of Group Theory to Quantum Physics Algebraic Aspects	36
L. O'RAIFEARTAIGH - Unitary Representations of Lie Groups in Quantum Mechanics	144
BERTRAM KOSTANT - On Certain Unitary Representations Which Arise From a Quantization Theory	237
I. T. TODOROV - Derivation and Solution of an Infinite-Component Wave Equation for the Relativistic Coulomb Problem	254
W. RÜHL - Tensor Operators for the Group $SL(2, \mathbb{C})$	279
G. A. GOLDIN and D. H. SHARP - Lie Algebras of Local Currents and Their Representations	300
ROBERT HERMANN - Infinite Dimensional Lie Algebras and Current Algebra . . .	312
List of Attendees	339