

Lecture Notes in Physics

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Stochastic Processes Formalism and Applications

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PREFACE

A winter school on STOCHASTIC PROCESSES - FORMALISM AND APPLICATIONS was held at the University of Hyderabad December 15 - 24, 1982. These proceedings are based on lectures delivered at the School. The aim was to introduce research workers at the pre- and post-doctoral levels to the basic concepts, techniques and applications of stochastic theories. Some of the discussions, such as those on the properties of Gaussian and Markov processes and on Langevin and Fokker-Planck equations, have by now become standard textbook material. It was decided, however, to include them in this volume in order to make it a self-contained source book or even a graduate level reference book on the subject. On the other hand, certain other topics, not normally dealt with in the context of stochastic processes (e.g. random matrices) also find place in the proceedings. Similarly, the important problem of localization of electronic states in disordered metals is included in view of its natural connection to diffusion, a paradigm of stochastic processes.

The lectures can be divided into two broad categories : (i) basic concepts and techniques, and (ii) applications. Under (i), in addition to fundamental ideas in stochastic processes, certain formal techniques involving projection operators in both stochastic differential equations and master equations, system-size expansions, continuous-time random-walk models, decay of metastable states, etc. are treated in detail. Two lectures on numerical techniques, an often neglected but useful aspect, are also presented here. The basic formalism covered here forms a natural setting for the applications discussed later. It is hoped that the reader will find the right kind of balance between techniques on the one hand and physical applications on the other.

The applications, of course, constitute the central motivating theme of the school. Here, such diverse and topical subjects as Brownian motion, nonequilibrium phase transitions, disordered systems, wave propagation in random media, line shapes in optics and other spectroscopy problems are discussed. For the sake of conciseness, only an outline of the mathematical steps is provided in some lectures. This lacuna is, however, filled-in with the aid of numerous references which the interested reader can look into.

It would not have been possible to organize the school without the financial and material support from the Department of Science and Technology, Government of India and the University of Hyderabad. We would like to record our gratitude to both these institutions. We thank Mr. A. Neela Kantam and Mr. P.C.P. Reddy for skilful typing of the manuscripts. We are greatly indebted to Ms. B. Aruna, Mr. N.V. Mukteswara Rao and Mr. D. Sridhara Rao for their invaluable assistance in the preparation of the proceedings.

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G.S. AGARWAL
S. DATTA GUPTA

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