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Andre N. Luiten (Ed.)

Frequency Measurement and Control

Advanced Techniques and Future Trends

With 169 Figures and 9 Tables

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Dr. Andre N. Luiten (Ed.)
Physics Department
University of Western Australia
Nedlands 6907 WA
Australia
Email: andre@physics.uwa.edu.au

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Preface

Throughout history, periodic phenomena have provided the most stable reference sources for mankind; these oscillators have acted as a stable ‘rule’ so that the most precise measurements could be made. In earlier times the best oscillators were found in some naturally occurring repetitive phenomena in Nature, while in more recent times mankind has developed artificial methods to better approximate the ideal oscillator. At several moments in our past, the discovery or development of a new type of oscillator with improved performance enabled a more refined measurement, and hence was the precipitating factor in an extension of our understanding of the Universe. For example, the use of pendulum-oscillators allowed the experimental verification of Galilean and Newtonian laws of mechanics. For a more recent example, one could consider the long-term observation of the Hulse-Taylor binary pulsar system. The observation of this astronomical oscillator confirmed many of the predictions of the Theory of General Relativity with a remarkable precision. In a few cases these oscillator-enabled measurements were a prelude to a revolution in our understanding, e. g. the observation of an excess precession in the perihelion of Mercury. The research described in this volume forms part of the continuing quest to improve the performance of oscillators. One can easily believe that some of our present understanding will need to be revised in response to the new vistas that will be glimpsed with the aid of this generation of oscillators.

The field of frequency control and measurement is, like many of the experimental sciences, a fusion of art and science. Many of the key techniques cannot be found in textbooks, but are instead passed down by word of mouth from researcher to student (or generously from researcher to researcher). This characteristic of the field is perhaps best exemplified by J. Hall’s “law” of frequency-standards’ improvement:

- one afternoon (3 hours) is sufficient to lock an oscillator to the center of some stabilizing resonance feature to within 1% of the width of the feature;
- three months is required to achieve 0.03%;
- thirty years or more is required to achieve 1 part in 10^6 of the resonance feature width.

Hall’s law has been (surprisingly) verified many times, in many laboratories, across the electromagnetic spectrum from the radio frequency to the optical domains, even though the scientific basis for the law is not so clear! One of my hopes for this volume is that it can provide an entry point to many of the exciting developments of the field. By presenting a collection of many of the disparate elements of the field, I hope that some of the flavor of the field, the ‘art’ if you like, will be communicated to the interested reader.

I hope also that the careful reader may note a challenge which is implied throughout this volume. The performance of a frequency standard is essentially determined by two factors: the width of the resonance feature to which the oscillator is locked, and how well the oscillator can be locked to this resonance feature. The most highly developed optical frequency standards are presently based on resonance features with Q-factors ($Q = \nu_0/\Delta\nu$ where ν_0 is the center frequency of the transition, and $\Delta\nu$ is the width of the transition) in the range of 10^{12}–10^{15}. In close accord with *Hall's* law, after 20 to 30 years of research the best optical standards have demonstrated frequency stabilization to parts in 10^3–10^5 of their resonance linewidth. However, as we will see later in this volume, Q-factors of up to 10^{23} have been observed in some experiments. Thus, one could easily expect that, by further development of stabilization techniques and by moving to higher Q-factor transitions, there are many orders of magnitude of improvement still to come. The first 10^{-17} frequency standard is awaiting your contribution!

The field of frequency measurement and control possesses one other peculiar characteristic: although the overriding goal of the experiment may be the development of better frequency standards or more precise frequency-measurement techniques, the actual specific methods used, and hurdles to be overcome, fall in many different fields of expertise – e.g. atomic, ion or quantum physics, optical engineering and physics, solid-state physics, electronics, microwave engineering and physics, laser physics, measurement science, or mechanical engineering. This spread of expertise has the disadvantage that many new, but highly relevant, discoveries appear in many different places in the scientific and engineering literature. The advantage of this broadness is, of course, the maintenance of the interest of the researcher. I hope that this volume may go some way towards providing a unified source of information on the field.

Finally, I wish to thank the many people who have made this collection of articles possible. First, I extend my gratitude to the many authors who so generously contributed their time to make a contribution to this volume. Writing an in-depth analysis of an area of research always takes time away from things one would rather be doing, and thus I am indebted to all of the expert authors for choosing to put this job at the top of their priority queue. I would like to thank John McFerran in particular for carefully rereading each of the manuscripts to look for oversights on my part, as well as to suggest many improvements. To finish, I must acknowledge the patience of Sylvia who has always understood the unreasonable hours I spend in the laboratory, and who has provided incredible support even while I did so.

University of Western Australia,
Perth, Australia
July 2000 *Andre N. Luiten*

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