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L. Ting R. Klein

Viscous Vortical Flows



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Authors

Lu Ting

Courant Institute of Mathematical Sciences
New York University, 251 Mercer Street
New York, NY 10012, USA

Rupert Klein

Institut für Technische Mechanik, RWTH Aachen
Templergraben 64, D-5100 Aachen, BRD

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Preface

These lecture notes are based upon a series of ten lectures given by L. Ting in commemoration of the 75th anniversary of the founding of the Aerodynamisches Institut, Rheinisch-Westfälische Technische Hochschule Aachen, in April 1988. Besides making some amplifications here and there and the addition of a few recent results, we retain the lecture style, the sequence and the spirit delivered there. No attempt was made to hide the authors' biases and interests.

The lecture notes are composed of four chapters preceded by an extensive introduction in which the contents of each chapter are summarized section by section.

In the first chapter we formulate the mathematical problem of a vortex-dominated viscous flow as an initial value problem for the incompressible Navier-Stokes (N-S) equations in an unbounded domain with an initial vorticity field. Some general properties of the solutions are presented, namely, the consistency conditions on the moments of vorticity, and their relationships to the far field representations of the vector and scalar potentials of velocity.

In the second chapter we study the motion of slender vortex filaments with small diffusive core structures, using the method of matched asymptotics. Details of the analyses were reported in a sequence of four papers which began in 1965 with the relatively simple two dimensional problem and ended in 1978 with the three dimensional problems of slender vortex filaments with large axial and circumferential velocities in their cores. Here we emphasize the scalings, or rather the intuition, leading to the general scheme of the asymptotic analysis and the physical interpretation of the solutions. Also we present some recent studies on the practical region of validity of the asymptotic solutions.

In the third chapter we study the merging of the core structures of vortices by numerical solution of the N-S equations. We explain our numerical schemes and how the asymptotic and numerical methods can complement each other in the solution for the entire flow field. In the last chapter we mention several new problem areas.

Finally we wish to thank Professor Egon Krause, director of the Aerodynamisches Institut, for his invitation to the commemoration lectures and his constant encouragement, without which this book could never have materialized. We are especially indebted to Professors Joseph B. Keller of Stanford University and Andrew Majda of Princeton University who read our preliminary manuscript and made many useful suggestions that were incorporated in the final text. We grate-

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New York, NY

L. Ting

November 9, 1990

R. Klein

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