

Lecture Notes in Physics

Edited by H. Araki, Kyoto, J. Ehlers, München, K. Hepp, Zürich
R. Kippenhahn, München, H.A. Weidenmüller, Heidelberg
J. Wess, Karlsruhe and J. Zittartz, Köln
Managing Editor: W. Beiglböck

286

R. Alicki
K. Lendi

Quantum Dynamical Semigroups
and Applications



Springer-Verlag

Berlin Heidelberg New York London Paris Tokyo

Authors

Robert Alicki

Institute of Theoretical Physics and Astrophysics
University of Gdańsk, PL-80-952 Gdańsk, Poland

Karl Lendi

Institute of Physical Chemistry, University of Zürich
CH-8057 Zürich, Switzerland

ISBN 3-540-18276-4 Springer-Verlag Berlin Heidelberg New York

ISBN 0-387-18276-4 Springer-Verlag New York Berlin Heidelberg

This work is subject to copyright. All rights are reserved, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, re-use of illustrations, recitation, broadcasting, reproduction on microfilms or in other ways, and storage in data banks. Duplication of this publication or parts thereof is only permitted under the provisions of the German Copyright Law of September 9, 1965, in its version of June 24, 1985, and a copyright fee must always be paid. Violations fall under the prosecution act of the German Copyright Law.

© Springer-Verlag Berlin Heidelberg 1987

Printed in Germany

Printing: Druckhaus Beltz, Hemsbach/Bergstr.;

Bookbinding: J. Schäffer GmbH & Co. KG., Grünstadt

2153/3140-543210

P R E F A C E

The original idea of publishing Lecture Notes on this topic arose spontaneously after invited talks given by the authors at the XXth Symposium of Theoretical Chemistry held in 1984 in Emmetten, Switzerland. As a result of many discussions we felt a real need for popularization of the unifying and fruitful, but apparently widely unknown, concept of complete positivity. The associated semigroup dynamics of open quantum systems covers so many different types of interesting irreversible processes that this theory should be made more easily accessible to a wider public. Our common desire to give a rather broad presentation of the subject soon caused the manuscripts to lose their original character of contributions to conference proceedings. During continuous attempts to improve upon the actual versions and to include more and more topics and the latest research results, much time elapsed, and now we can say that many conference talks, many special lecture courses given at the universities of Gdańsk and Zürich and various recently published and also unpublished papers, as well as discussions with colleagues, contributed to the final versions.

Each author's contribution reflects strongly the area of his own research activities, but we could not resist the challenge of a joint presentation as a synthesis of a whole with a broad spectrum ranging from the abstract theory up to very concrete applications, even to recent experiments.

We hope that this book provides a valuable survey of many relevant aspects of the quantum theory of irreversible processes.

R. Alicki
Gdańsk, June 1987

K. Lendi
Zürich, June 1987

Contents

GENERAL THEORY AND APPLICATIONS TO

UNSTABLE PARTICLES by R. Alicki

Part A. GENERAL THEORY

I.	Introduction	1
II.	Completely positive dynamical semigroups	
II.1	Reduced dynamics	2
II.2	Completely positive maps	5
II.3	Generalized H-theorem	8
II.4	Generators of quantum dynamical semigroups	12
II.5	How to construct generators?	17
III.	Hamiltonian models and Markovian approximation	
III.1	Generalized master equation	19
III.2	Weak coupling limit	21
III.3	Low density limit	26
III.4	Heat bath, detailed balance and return to equilibrium	31
III.5	Singular coupling limit	36
IV.	Extensions of the formalism	
IV.1	Nonconservative dynamical semigroups	38
IV.2	Time-dependent generators	40
IV.3	Nonlinear quantum dynamical semigroups	41
IV.4	Discrete quantum Boltzmann equation	43
IV.5	Nonlinear Schroedinger equation	45
V.	A system of N 2-level atoms	
V.1	The Hamiltonian of the system	48
V.2	The Markovian master equation	50
V.3	Return to equilibrium and superradiance	54

Part B. QUANTUM DYNAMICAL SEMIGROUPS FOR UNSTABLE PARTICLES

VI.	Introduction	58
VII.	Damped and Pumped Quantum Harmonic Oscillator	
VII.1	Derivation of Markovian master equation	59
VII.2	Birth and death process, kinetic equation	61
VII.3	Explicit solutions	62

VIII. Models of unstable particles	
VIII.1 Fock spaces and quantum fields	64
VIII.2 Construction of Markovian master equation	68
VIII.3 Single-particle description	70
VIII.4 Explicit solutions	71
VIII.5 Hamiltonian models of unstable particles	77
VIII.6 Relativistic unstable particles	80
Appendices	
A.1 Banach spaces $\mathcal{B}(\mathcal{H})$ and $\mathcal{T}(\mathcal{H})$	85
A.2 One-parameter semigroups	87
A.3 Quantum correlation functions	90
References	92

N-LEVEL SYSTEMS AND APPLICATIONS TO
SPECTROSCOPY by K. Lendi

I.	Introduction	95
II.	General structure of quantum Markovian master equations for N-level systems	
II.1	The Kossakowski-generator of time-evolution	97
II.2	Positive-semidefiniteness of the relaxation matrix.....	99
II.3	Complete orthonormal matrix sets	101
II.4	Coherence-vector formulation	105
II.5	Relaxing semigroups	112
III.	Two-level systems: Generalized magnetic or optical Bloch-equations	
III.1	Details of the full relaxation equations for static external fields	117
III.2	Alternating external fields and constant relaxation	122
III.3	Modified lineshapes and free induction decay	125
IV.	Three-level systems	
IV.1	General equations	131
IV.2	Bloch-equations for decaying systems	132
V.	Comparison with common versions of master equations	
V.1	General considerations	139
V.2	Equations for spontaneous emission	140
V.3	Equations of Lamb-type	143
VI.	Open quantum systems with non-constant relaxation in time-dependent external fields	
VI.1	Modifications of the original semigroup generator	146
VI.2	A model with field-strength-dependent relaxation	149

VII.	Determination of relaxation parameters from first principles	
VII.1	Relationship between Kossakowski- and Davies-generators	152
VII.2	A model for spin-relaxation by spin-waves	159
VIII.	Entropy and irreversibility	
VIII.1	Entropy production	166
VIII.2	Measure of irreversibility	173
IX.	Conclusion	182
Appendices		
A:	Generators and structure constants for $SU(N)$, $N = 2,3,4$	184
B:	Eigenvalues of the general two-level evolution matrix	188
C:	Elements of the time-dependent two-level evolution matrix	190
References	191