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# Advances in Polaron Physics

With 43 Figures

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## Preface

While basic features of polarons were well recognized a long time ago and have been described in a number of review papers and textbooks, interest in the role of electron–phonon interactions and polaron dynamics in different materials has recently gone through a vigorous revival. Electron–phonon interactions have been shown to be relevant in many inorganic and organic semiconductors and polymers, colossal magnetoresistance oxides, and transport through nanowires and quantum dots also often depends on vibronic displacements of ions. These interactions presumably play a role in high-temperature superconductors as well. The continued interest in polarons extends beyond the physical description of advanced materials. The field has been a testing ground for analytical, semi-analytical, and numerical techniques, such as path integrals, strong-coupling perturbation expansion, advanced variational methods, exact diagonalization, Quantum Monte Carlo, and other techniques. This book reviews some recent developments in the field of polarons, starting with the basics and covering a number of active directions of research. Single- and multipolaron theories have offered more insight into colossal magnetoresistance and in a broad spectrum of physical properties of structures with reduced dimension and dimensionality such as transport, optical absorption, Raman scattering, photoluminescence, magneto-optics, etc. While nobody – at present – has a final theory of high-temperature superconductivity, we discuss one alternative (polaronic) route. We have benefited from discussions with many experts in the field. While writing this book and at some earlier stage of our studies, discussions with A. Andreev, S. Aubry, I. Bozovic, A. Bratkovsky, F. Brosens, P. Edwards, J. Bonca, V. Cataudella, G. De Filippis, Yu. Firsov, V. Fomin, R. Evrard, H. Fehske, V. Gladilin, J. Hague, M. Hohenadler, G. Iadonisi, V. Kabanov, E. Kartheuser, S. Klimin, P. Kornilovitch, L. Lemmens, W. von der Linden, D. Mihailovic, A. Mishchenko, G. Papadopoulos, F. Peeters, J. Samson, M. Stoneham, J. Tempere, and P. Zhao were especially helpful.

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