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A Primer for Chiral Perturbation Theory

 Springer

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Preface

Chiral perturbation theory (ChPT) is the effective field theory of quantum chromodynamics (QCD) at energies well below typical hadron masses. This means that it is a systematic and model-independent approximation of QCD, based on the symmetries of the underlying theory and general principles of quantum field theory. Starting from early work on the interaction of pions, ChPT has grown to become a valuable tool to analyze and interpret a host of low-energy experiments involving the lowest-mass meson and baryon octets and decuplets. The application to $\pi\pi$ scattering and pion photoproduction are just two of the large number of remarkable successes of ChPT.

This monograph is based on lectures on chiral perturbation theory given by one of us (S.S.) on various occasions, supplemented with additional material. It is aimed at readers familiar with elementary concepts of field theory and relativistic quantum mechanics. The goal of these lecture notes is to provide a *pedagogical introduction* to the basic concepts of chiral perturbation theory (ChPT) in the mesonic and baryonic sectors. We therefore also derive and explain those aspects that are considered well known by “experts.” In particular, we often include intermediate steps in derivations to illuminate the origin of our final results. We have also tried to keep a reasonable balance between mathematical rigor and illustrations by means of simple examples. Numerous exercises throughout the text cover a wide range of difficulty, from very easy to quite difficult and involved. Ideally, at the end of the course, the reader should be able to perform simple calculations in the framework of ChPT and to read the current literature. Solutions to all exercises are provided for readers to check their own work.

These lecture notes include the following topics: [Chapter 1](#) deals with QCD and its global symmetries in the chiral limit, explicit symmetry breaking in terms of the quark masses, and the concept of Green functions and Ward identities reflecting the underlying chiral symmetry. In [Chap. 2](#), the idea of a spontaneous breakdown of a global symmetry is discussed and its consequences in terms of the Goldstone theorem are demonstrated. [Chapter 3](#) deals with mesonic chiral perturbation theory. The principles entering the construction of the chiral Lagrangian are outlined and a number of elementary applications are discussed. In [Chap. 4](#), these methods

are extended to include the interaction between Goldstone bosons and baryons in the single-baryon sector. [Chapter 5](#) discusses more advanced applications and topics that are closely related to chiral perturbation theory.

This work is not intended as a comprehensive review of the status of chiral perturbation theory. This also means that we cannot cite all of the vast literature, especially on advanced applications. Readers interested in the present status of applications are encouraged to consult the numerous available lecture notes, review articles, and conference proceedings. A list of suggested references is provided at the end of [Chap. 5](#).

While the number of people who have contributed to our understanding of the topics discussed in this monograph is too large to acknowledge each of them individually, we would like to thank H.W. Fearing, J. Gegelia, H.W. Grießhammer, and D. R. Phillips for numerous interesting and stimulating discussions that have most directly influenced us. We are grateful to A. Neiser for the careful reading of and helpful comments on the manuscript. We would also like to thank all students who participated in previous classes on ChPT and gave important feedback. The support and patience of our editor C. Caron is gratefully acknowledged. S.S. would like to thank M. Hilt for extensive technical support. M.R.S. would like to thank the Lattice and Effective Field Theory group at Duke University for their hospitality. This work was carried out in part with financial support from the Center for Nuclear Studies at the George Washington University, National Science Foundation CAREER award PHY-0645498, and US-Department of Energy grant DE-FG02-95ER-40907.

Mainz and Columbia, SC, April 2011

Stefan Scherer
Matthias R. Schindler

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