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Eric Vittoz

# Low-Power Crystal and MEMS Oscillators

## The Experience of Watch Developments

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**To my wife Monique**

# Contents

# Preface

In the early 60s, the watchmaking industry realized that the newly invented integrated circuit technology could possibly be applied to develop electronic wristwatches. But it was immediately obvious that the precision and stability required for the time base could not be obtained by purely electronic means. A mechanical resonator had to be used, combined with a transducer. The frequency of the resonator had to be low enough to limit the power consumption at the microwatt level, but its size had to be compatible with that of the watch. After unsuccessful results with metallic resonators at sonic frequencies, efforts were concentrated on reducing the size of a quartz crystal resonator. Several solutions were developed until a standard emerged with a thin tuning fork oscillating at 32 kHz and fabricated by chemical etching. After first developments in bipolar technology, CMOS was soon identified as the best choice to limit the power consumption of the oscillator and frequency divider chain below one microwatt. Low-power oscillator circuits were developed and progressively optimized for best frequency stability, which is the main requirement for timekeeping applications. More recent applications to portable communication devices require higher frequencies and a limited level of phase noise. Micro-electro-mechanical (MEM) resonators have been developed recently. They use piezoelectric or electrostatic transduction and are therefore electrically similar to a quartz resonator.

The precision and stability of a quartz is several orders of magnitude better than that of integrated electronic components. Hence, an ideal oscillator circuit should just compensate the losses of the resonator to maintain its oscillation on a desired mode at the desired level, without affecting the frequency or the phase of the oscillation. Optimum designs aim at approaching this ideal case while minimizing the power consumption.

This book includes the experience accumulated along more than 30 years by the author and his coworkers. The main part is dedicated to variants of the Pierce oscillator most frequently used in timekeeping applications. Other forms of oscillators that became important for RF applications have been added, as well as an analysis of phase noise. The knowledge is formalized in an analytical manner, in order to highlight the effect and the importance of the various design parameters. Computer simulations are limited to particular examples but have been used to crosscheck most of the analytical results.

Many collaborators of CEH (Centre Electronique Hologer, Watchmakers Electronic Center), and later of CSEM, have contributed to the know-how described in this book. Among them, by alphabetic order, Daniel Aebischer, Luc Astier, Serge Bitz, Marc Degrauwe, Christian Enz, Jean Fellrath, Armin Frei, Walter Hammer, Jean Hermann, Vincent von Kaenel, Henri Oguey, and David Ruffieux. Special thanks go to Christian Enz for the numerous discussions about oscillators and phase noise during the elaboration of this book.

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# Symbols

**Table 0.1** Symbols and their definitions.

| Symbol | Description | Reference |
|---|---|---|
| $a$ | Power factor of the flicker noise current | (6.71) |
| $A$ | Normalized transconductance in series resonance oscillator | (6.108) |
| $B$ | Normalized bandwidth in series resonance oscillator | (6.108) |
| $C_a, C_b$ | Functional capacitors | Fig. 6.37 |
| $C_D$ | Capacitance between drains | Fig. 6.1 |
| $C_L$ | Load capacitance in series resonance oscillator | Fig. 6.16 |
| $C_m$ $(C_{m,i})$ | Motional capacitance (of mode $i$) | Fig. 2.2 |
| $C_P$ | Total parallel capacitance of the resonator | (2.22) |
| $C_s$ | Series connection of $C_1$ and $C_2$ | (4.9) |
| $C_S$ | Capacitance between sources | Fig. 6.1 |
| $C_0$ | Parallel capacitance of the dipole resonator | (2.1) |
| $C_1$ | Total gate-to-source capacitance | Fig. 4.1 |
| $C_2$ | Total drain-to-source capacitance | Fig. 4.1 |
| $C_3$ | Total capacitance across the motional impedance | Fig. 2.2 |
| $E_m$ | Energy of mechanical oscillation | (2.23) |
| $f$ | Frequency | |
| $f_m$ | Motional resonant frequency | (4.140) |
| $f_s$ | Frequency of stable oscillation | (3.24) |
| $\mathrm{f}_s(m_v)$ | Fundamental function in strong inversion | (6.37) |
| $\mathrm{f}_w(v_{in})$ | Fundamental function in weak inversion | (6.30) |
| $F_a$ | Flicker noise current constant | (6.71) |
| $G_a$ | Reference conductance for the flicker noise current | (6.71) |
| $G_{ds}$ | Residual output conductance in saturation | (3.57) |
| $G_m$ | Gate transconductance of a transistor | (3.53) |

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| **Symbol** | **Description** | **Reference** |
|---|---|---|
| $G_{ms}$ | Source transconductance of a transistor | (3.49) |
| $G_{md}$ | Drain transconductance of a transistor | (3.49) |
| $G_{mcrit}$ | Critical transconductance for oscillation | Fig. 4.4 |
| $G_{mcrit0}$ | Critical transconductance for lossless circuit | Fig. 4.6 |
| $G_{mlim}$ | Limit transconductance in series resonance oscillator | (6.109) |
| $G_{mmax}$ | Maximum possible transconductance for oscillation | Fig. 4.4 |
| $G_{mopt}$ | Optimum value of transconductance | Fig. 4.4 |
| $G_{m(1)}$ | Transconductance for the fundamental frequency | (4.54) |
| $G_{vi}$ | Transconductance of the regulator | (5.52) |
| $\mathrm{h}_s(m_i)$ | Transconductance function in strong inversion | (6.142) |
| $\mathrm{I}_{\mathrm{B0}}(x)$ | Modified Bessel function of order 0 | (4.59) |
| $\mathrm{I}_{\mathrm{B1}}(x)$ | Modified Bessel function of order 1 | (4.61) |
| $I_c$ | Circuit current | Fig. 3.1 |
| $I_{cs}$ | Value of $I_c$ at stable oscillation | (3.6) |
| $I_D$ | Drain current | Fig. 3.10 |
| $I_{D0}$ | DC component of drain current | (5.1.2) |
| $I_{D(1)}$ | Fundamental component of $I_D$ | Fig. 4.13 |
| $I_F$ | Forward component of drain current | (3.40) |
| $I_m$ | Motional current | Fig. 2.2 |
| $I_{ms}$ | Value of $I_m$ at stable oscillation | (3.6) |
| $I_R$ | Reverse component of drain current | (3.40) |
| $I_{spec}$ | Specific current of a transistor | (3.41) |
| $I_0$ | Bias current of the oscillator | Fig. 4.13 |
| $I_{0start}$ | Start-up value of bias current | (5.45) |
| $I_{0crit}$ | Critical value of bias current $I_0$ | Fig. 4.14 |
| $I_{0critmin}$ | Critical current in weak inversion | (4.64) |
| $I_1$ | Complex value of the sinusoidal drain current | 6.3.2.3 |
| $IC$ | Inversion coefficient of a transistor | (3.45) |
| $IC_0$ | Inversion coefficient at $I_0 = I_{0crit}$ | (4.72) |
| $k_c$ | Capacitive attenuation factor | (4.69) |
| $K_f$ | Flicker noise voltage constant of a transistor | (3.62) |
| $K_{fi}$ | Flicker noise current function | (3.36) |
| $K_{fv}$ | Flicker noise voltage function | (3.35) |
| $K_g$ | Transconductance ratio | Fig. 6.37 |
| $K_i$ | Mirror ratio in the regulator | Fig. 5.9 |
| $K_{iv}$ | Gain parameter of $\lvert V_1\rvert(I_{D0})$ | (5.24) |
| $K_l$ | Level of specific current | (6.92) |
| $K_m$ | Margin factor | (4.17) |
| $K_r$ | Ratio of transfer parameters | Fig. 5.4 |

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| **Symbol** | **Description** | **Reference** |
|---|---|---|
| $K_s$ | Ratio of specific currents | (6.82) |
| $K_t$ | Transconductance ratio | (6.175) |
| $K_w$ | Width ratio in the regulator | (5.43) |
| $L_m$ ($L_{m,i}$) | Motional inductance (of mode $i$) | Fig. 2.2 |
| $m_i$ | Index of current modulation | (6.133) |
| $m_v$ | Index of voltage modulation | (4.122) |
| $m_{vd}$ | Index of voltage modulation for a differential pair | (6.35) |
| $M$ | Figure of merit | (2.9) |
| $M_D$ | Figure of merit of the resonator used as a dipole | (2.22) |
| $M_L$ | Figure of merit of the resonator used as a loaded dipole | (6.7) |
| $M_0$ | Intrinsic figure of merit of the resonator | (2.10) |
| $n$ | Slope factor of a transistor | (3.40) |
| $p$ | Frequency pulling | (2.7) |
| $p_c$ | Frequency pulling at critical condition for oscillation | (3.10) |
| $p_{pa}$ | Frequency pulling at parallel resonance | (2.15) |
| $p_s$ | Frequency pulling at stable oscillation | (3.7) |
| $p_{se}$ | Frequency pulling at series resonance | (2.14) |
| $P_m$ | Power dissipated in the resonator | (2.24) |
| $Q$ ($Q_i$) | Quality factor (of mode $i$) | (2.3) |
| $Q_b$ | Quality factor of the bias circuit | (5.12) |
| $R_{iv}$ | Slope of the amplitude $\|V_1\|(I_0)$ | Fig. 5.3 |
| $R_L$ | Load resistance | Fig. 6.16 |
| $R_m$ ($R_{m,i}$ ) | Motional resistance (of mode $i$) | Fig. 2.2 |
| $R_n$ | Negative resistance of the circuit | (3.2) |
| $R_{n0}$ | Value of $R_n$ for the linear circuit | (3.12) |
| $s_{vi}$ | Normalized slope of the regulator | Fig. 5.10 |
| $s_{iv}$ | Normalized slope of the amplitude | Fig. 4.17 |
| $S_{I_n^2}$ | Current noise spectrum | (4.107) |
| $S_{I_{nD}^2}$ | Drain current channel noise spectrum | (3.59) |
| $S_{I_{nL}^2}$ | Loop Current noise spectrum | (3.27) |
| $S_{V_n^2}$ | Voltage noise spectrum | (4.107) |
| $S_{V_{nG}^2}$ | Gate voltage flicker noise spectrum | (3.62) |
| $S_{\phi_n^2}$ | Phase noise power spectrum | (3.29) |
| $t$ | Time | |
| $U_T$ | Thermodynamic voltage | (3.40) |
| $V$ | Voltage across the resonator | Fig. 2.2 |
| $V_B$ | Supply voltage (battery voltage) | Fig. 5.1 |
| $v_c$ | Value of $\|V_c\|$ normalized to $nU_T$ | (6.130) |
| $V_c$ | Control voltage of a transistor | (6.129) |

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| **Symbol** | **Description** | **Reference** |
|---|---|---|
| $V_D$ | Drain voltage | Fig. 3.10 |
| $V_{Dsat}$ | Saturation value of drain voltage | (3.46) |
| $v_e$ | Normalized effective DC gate voltage | (4.57) |
| $V_G$ | Gate voltage | Fig. 3.10 |
| $V_{G0}$ | DC component of gate voltage | (4.52) |
| $v_{in}$ | Value of $\|V_{in}\|$ normalized to $nU_T$ | (6.31) |
| $V_{in}$ | Differential input voltage | Fig. 6.5 |
| $V_M$ | Channel length modulation voltage | (3.57) |
| $V_n$ | Open-loop noise voltage of the circuit | 3.7.1 |
| $V_S$ | Source voltage | Fig. 3.10 |
| $V_{T0}$ | Threshold voltage of a transistor | (3.40) |
| $V_{(1)}$ | Complex value of fundamental component of $V$ | (3.1) |
| $v_1$ | value of $\|V_1\|$ normalized to $nU_T$ | (4.57) |
| $V_1$ | Complex value of gate-to-source voltage | Fig. 4.8 |
| $V_2$ | Complex value of drain-to-source voltage | Fig. 4.8 |
| $V_3$ | Complex value of drain-to-gate voltage | Fig. 4.8 |
| $Z_c$ | Impedance of the linear circuit | (3.8) |
| $Z_{c(1)}$ | Circuit impedance for fundamental frequency | (3.1) |
| $Z_{c0}$ | Circuit impedance without parallel capacitance | (6.3) |
| $Z_D$ | Impedance between drains | Fig. 6.3 |
| $Z_L$ | Load impedance | Fig. 6.16 |
| $Z_m$ ($Z_{m,i}$) | Motional impedance (of mode $i$) | Fig. 2.2 |
| $Z_p$ | Total parallel impedance | (2.12) |
| $Z_S$ | Impedance between sources | Fig. 6.3 |
| $Z_1$ | Total gate-to-source impedance | Fig. 4.3 |
| $Z_2$ | Total drain-to-source impedance | Fig. 4.3 |
| $Z_3$ | Total drain-to-gate impedance | Fig. 4.3 |
| $\alpha$ | Ratio of critical transconductance | (4.98) |
| $\alpha_i$ | Noise current modulation function | Fig. 3.9 |
| $\alpha_v$ | Noise voltage modulation function | Fig. 3.9 |
| $\alpha_0$ | Value of $\alpha$ for the lossless case | (4.99) |
| $\beta$ | Transfer parameter of a transistor | (3.44) |
| $\Delta\omega$ | Noise frequency offset | (3.28) |
| $\varepsilon_{max}$ | Maximum relative mismatch | (6.15) |
| $\varepsilon_0$ | Permittivity of free space | (2.27) |
| $\gamma$ | Noise excess factor of the oscillator | Fig. 3.7 |
| $\gamma_t$ | Channel noise excess factor of a transistor | (3.60) |
| $\Gamma_i$ | Effective impulse sensitivity function for noise current | (3.34) |
| $\Gamma_v$ | Effective impulse sensitivity function for noise voltage | (3.32) |

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| **Symbol** | **Description** | **Reference** |
|---|---|---|
| $\tau$ | Time constant of oscillation growth | (3.16) |
| $\tau_0$ | Start-up value of $\tau$ | (3.15) |
| $\omega$ | Approximate angular frequency of oscillation | |
| $\omega_m$ ($\omega_{m,i}$) | angular frequency of resonance (of mode $i$) | (2.2) |
| $\omega_n$ | Angular frequency at which noise is considered | (4.106) |
| $\omega_s$ | Angular frequency at stable oscillation | (3.24) |
| $\Omega_{civ}$ | Cut-off angular frequency of $\|V_1\|(I_{D0})$ | (5.26) |
| $\Omega_0$ | Resonant angular frequency of bias circuit | (5.10) |
| $\Omega_1$ | Unity gain frequency of the regulation loop | (5.71) |