

Lectures on Numerical Methods

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*Lectures on
Numerical Methods*

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PREFACE

The course of lectures on numerical methods (part I) given by the author to students in the numerical third of the course of the mathematics-mechanics department of Leningrad State University is set down in this volume. Only the topics which, in the opinion of the author, are of the greatest value for numerical methods are considered in this book. This permits making the book comparatively small in size, and, the author hopes, accessible to a sufficiently wide circle of readers. The book may be used not only by students in daily classes, but also by students taking correspondence courses and persons connected with practical computation who desire to improve their theoretical background.

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The Author