Parametric Continuation and Optimal Parametrization in Applied Mathematics and Mechanics

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Preface

A decade has passed since *Problems of Nonlinear Deformation*, the first book by E.I. Grigoliuk and V.I. Shalashilin was published. That work gave a systematic account of the parametric continuation method. Ever since, the understanding of this method has sufficiently broadened. Previously this method was considered as a way to construct solution sets of nonlinear problems with a parameter. Now it is clear that one parametric continuation algorithm can efficiently work for building up any parametric set. This fact significantly widens its potential applications.

A curve is the simplest example of such a set, and it can be used for solving various problems, including the Cauchy problem for ordinary differential equations (ODE), interpolation and approximation of curves, etc. Research in this area has led to exciting results. The most interesting of such is the understanding and proof of the fact that the length of the arc calculated along this solution curve is the optimal continuation parameter for this solution.

We will refer to the continuation solution with the optimal parameter as the best parametrization and in this book we have applied this method to variable classes of problems: in chapter 1 to non-linear problems with a parameter, in chapters 2 and 3 to initial value problems for ODE, in particular to stiff problems, in chapters 4 and 5 to differential-algebraic and functional differential equations.

The fact that the transition to the best parameter in the Cauchy problem for ODE of normal form can be made using an analytical transformation, which we will call λ-transformation, has surprised even us.

Another notable result, which we discuss in chapter 6, is the development of a general approach to applications of the best parametrization in parametric approximation problems.

In chapter 7 we will consider potential applications of parametric continuation to building more complex one-parametric sets, i.e. solution sets for non linear boundary problems for ODE with a parameter.

Finally, in chapter 8, we will show how to use the best parametrization for the continuation of a solution in a neighborhood of singularities.

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